

Class 5: Wednesday September 10

Modifying the S-I-R Model

Given

$$S' = -0.00001SI$$

$$I' = 0.00001SI - (2/25)I$$

$$R' = (2/25)I$$

$$S_0 = 35000 \quad I_0 = 100 \quad R_0 = 4900$$

GROUPWORK:

1. During class today, we will try to develop IVPs for the following situations which modify the given S-I-R model above.

a. **Vaccination.** A modification of the original SIR model above after a partially successful vaccine is given to the population which cuts the infectiousness down to one quarter of its present infectiousness.

b. **Improve treatment.** A modification of the original SIR model after a treatment is discovered which reduces the time one is sick to 3 days.

c. **Immunity Loss.** A modification of the original SIR model so that 1 out of every 200 persons who recover become susceptible again.

- d. **Death.** A modification of the initial SIR model so that 1 out of every 30 persons who are infected dies, the rest recover.

Asymptotic Analysis

2. Consider the generic S-I-R model with parameters a , b and c . What happens after the epidemic has been running through the population for a very very long time? In other words, what happens as $t \rightarrow \infty$? What is $S_\infty = \lim_{t \rightarrow \infty} S$? $I_\infty = \lim_{t \rightarrow \infty} I$? $R_\infty = \lim_{t \rightarrow \infty} R$?

- a. Show that $(I + S - (b/a) \cdot \ln S)' = 0$

b. Therefore show that $S_\infty - \frac{b}{a} \ln(S_\infty) = I_0 + S_0 - \frac{b}{a} \ln(S_0)$

- c. What do these results above tell us about the possibility of someone never getting the disease? In other words, what is the value of S_∞ ?