Math 118 Fall 2003  ADVANCED PLACEMENT CALCULUS

Quiz 2  DUE: MON. SEP. 15

Name: ____________________

Date: ____________________  Friday, September 12, 2003  Ron Buckmi
Time Begun: ________________  
Time Ended: ________________

Topic covered: Euler’s Method and a System of ODEs

The idea behind the quiz is for you to illustrate your ability to deal with coupled ordinary differential equations and apply Euler’s Method to such a system.

Reality Check:

EXPECTED SCORE : __________/10  ACTUAL SCORE : __________/10

Instructions:

1. Once you open the quiz, you have 30 minutes to complete it.

2. You may not use the book or any of your class notes, but you may use a calculator. You must work alone.

3. If you use your own paper, please staple it to the quiz before coming to class. If you don’t have a stapler, buy one.

4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.

5. Relax and enjoy....

6. This quiz is due on Monday, September 15, at the beginning of class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.
SHOW ALL YOUR WORK

We will consider the following rate equations and initial conditions on the interval \([0, 1]\).

\[
S' = C, \quad C' = -S
\]

\[
S(0) = 0, \quad C(0) = 1
\]

a. \((6 \text{ points.})\) Use Euler’s method with \(\Delta t = 0.5\) to estimate the solutions \(S(t)\) and \(C(t)\) of the above initial value problem. Fill in the following table, which will help you find the approximating functions \(\hat{S}(t)\) and \(\hat{C}(t)\), which approximate the functions \(S(t)\) and \(C(t)\) which exactly solve the IVP.

<table>
<thead>
<tr>
<th>t</th>
<th>S</th>
<th>C</th>
<th>S'</th>
<th>C'</th>
<th>(\Delta S)</th>
<th>(\Delta C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/2</td>
<td></td>
<td></td>
<td>XXXXX</td>
<td>XXXXX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>XXXXX</td>
<td>XXXXX</td>
<td>XXXXX</td>
<td>XXXXX</td>
</tr>
</tbody>
</table>

Note: You should not need to use a calculator for this problem, but if you must use your calculator, DO NOT round off any decimal points.

b. \((2 \text{ points.})\) Show that, from the IVP alone, we can tell that the functions \(S(t)\) and \(C(t)\) obey the expression \(S^2 + C^2 = 1\) (HINT: Differentiate this expression with respect to time and use information from the differential equations and the initial condition.)

c. \((2 \text{ points.})\) Show that \(S(t) = \sin(t)\), \(C(t) = \cos(t)\) are the exact solutions to the IVP.