NAMES: ______________________________________

Lab 9: To Converge Or Not To Converge, That Is The Question

Introduction

This lab is designed to develop another test for convergence of a series. The first section is devoted to developing the test, and the second section is devoted to practicing all the tests of convergence.

For the following 6 series, use the tests for convergence that we have studied to determine whether the following series converge or not. Be sure to explain what test you are using and to show your work.

At the same time, we will develop a new test for convergence by gathering evidence from these six examples. Each of the six series has the form

$$\sum_{k=1}^{\infty} a_k$$

where $a_k \geq 0$. You are asked to identify the general term a_k and then to determine the value of L where

$$L = \lim_{k\to\infty} \sqrt[k]{a_k} = \lim_{k\to\infty} (a_k)^{1/k}.$$

Note: In determining the value of this limit L, it might help to know that

$$\lim_{k\to\infty} \sqrt[k]{k} = \lim_{k\to\infty} (k)^{1/k} = 1.$$

Take a moment now to check this out with your calculator: calculate the following: $10^{0.1}$; $100^{0.01}$; $1,000^{0.001}$; $10,000^{0.0001}$; $100,000^{0.00001}$. Convinced?

What about the limit of the kth root of a constant c? Assume $c > 1$. Then

$$\lim_{k\to\infty} \sqrt[k]{c} = \lim_{k\to\infty} (c)^{1/k} = \underline{\qquad\qquad}.$$

1. $\sum_{k=1}^{\infty} \frac{6}{e^k}$

Circle one: The series converges / does not converge. $a_k =$ ______________ $L =$ ______________
(Show your work for the limit L below.)

2. $\sum_{k=1}^{\infty} \frac{7}{9k^k}$

Circle one: The series converges / does not converge. $a_k =$ ______________ $L =$ ______________
(Show your work for the limit L below.)

3. $\displaystyle\sum_{k=1}^{\infty} \frac{4}{k^4}$

Circle one: The series converges / does not converge. $a_k =$ ______________ $L =$ ______________
(Show your work for the limit L below.)

4. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

Circle one: The series converges / does not converge. $a_k =$ ______________ $L =$ ______________
(Show your work for the limit L below.)

5. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{3^k}$

Circle one: The series converges / does not converge. $a_k =$ ______________ $L =$ ______________
(Show your work for the limit L below.)

6. $\displaystyle\sum_{k=1}^{\infty} \frac{4^k}{k^4}$

Circle one: The series converges / does not converge. $a_k =$ ______________ $L =$ ______________
(Show your work for the limit L below.)

Now let's summarize your results:

Series	circle one	value of L	Fill in with $<$, $=$ or $>$
1.	converges / does not converge		L _______ 1
2.	converges / does not converge		L _______ 1
3.	converges / does not converge		L _______ 1
4.	converges / does not converge		L _______ 1
5.	converges / does not converge		L _______ 1
6.	converges / does not converge		L _______ 1

From the evidence you have gathered in the table, we'll make some conjectures. For an infinite series of the form $\sum a_k$ where $a_k \geq 0$, we define $L = \lim \sqrt[k]{a_k}$ as $k \to \infty$.

- If $L < 1$, what can be said about $\sum a_k$?

- If $L > 1$, what can be said about $\sum a_k$?

- If $L = 1$, what can be said about $\sum a_k$?

Now we have a new test for convergence of a series to play with! It is referred to as the **Root Test**.

Practice Makes Perfect

The Root Test: Given the series $\sum_{k=1}^{\infty} a_k$ with $a_k \geq 0$, suppose that $\lim_{k\to\infty} \sqrt[k]{a_k} = L$. Then

- If $L < 1$, then $\sum_{k=1}^{\infty} a_k$ converges.
- If $L > 1$, then $\sum_{k=1}^{\infty} a_k$ does not converge.
- If L=1, the test is inconclusive.

Adding the root test to your repertoire of convergence tests, test each of the following series for convergence. Be sure to indicate the test that you are using to determine convergence and to show your work.

a. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(\ln k)^k}$

b. $\displaystyle\sum_{k=1}^{\infty} (\frac{2}{k})^k$

c. $\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{k}{3k+1}$

d. $\displaystyle\sum_{k=1}^{\infty}\frac{7^k}{k^2}$

e. $\displaystyle\sum_{k=1}^{\infty}ke^{-k}$

f. $\displaystyle\sum_{k=1}^{\infty}\frac{k^2}{(k^3+1)^2}$

g. $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^{1.1}}$

h. $\sum_{k=1}^{\infty} e^{-k^2}$

i. $\sum_{k=1}^{\infty} \frac{k^2 3^k}{k!}$

j. $\sum_{k=0}^{\infty} (-1)^k \frac{1}{1+\sqrt{k}}$

Assignment.

Hand in ONE neat copy of this lab handout per group. The lab assignment is due on **Thursday, December 4**.