Class 34: Monday December 2

Fourier Series

Fourier Series

In general, a Fourier Series is used to approximate a function $f(t)$ with period $[0, T]$

$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(\frac{2k\pi}{T}t) + \sum_{k=1}^{\infty} b_k \sin(\frac{2k\pi}{T}t)$$

where

$$\begin{aligned} a_0 &= \frac{1}{T}\int_0^T f(t)\ dt \\ a_k &= \frac{2}{T}\int_0^T f(t)\ \cos(\frac{2k\pi}{T}t)\ dt \\ b_k &= \frac{2}{T}\int_0^T f(t)\ \sin(\frac{2k\pi}{T}t)\ dt \end{aligned}$$

This usually involves a fair amount of integration.

EXAMPLE

$f(x) = \begin{cases} x & \text{if } -1 \leq x \leq 0 \\ -x & \text{if } 0 < x < 1 \end{cases}$

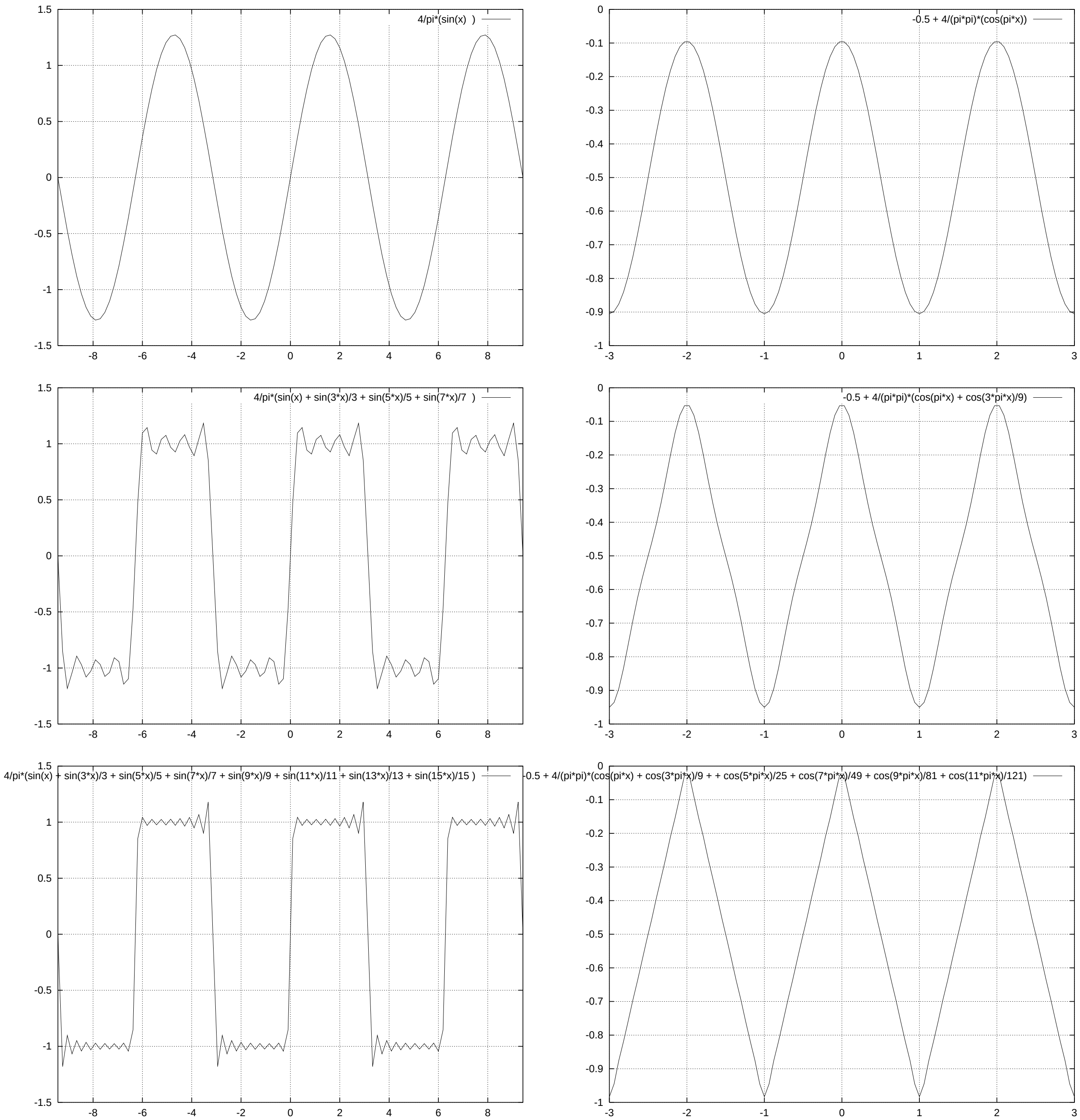
1. Sketch the graph of $f(x)$ below.

2. Find the zeroth degree Fourier polynomial for $f(x)$.

3. How would you show the general form of the Fourier series for $f(x)$ is $F_\infty(x) = -\frac{1}{2} + \sum_{k=1}^{\infty} \frac{2 - 2\cos(k\pi)}{(k\pi)^2} \cos(k\pi x)$.

4. For what values of x will the infinite series converge? Which test would you use?

The fourier series from the quiz looks like the graphs in the lefthand column:
The figures show $F_1(x)$, $F_7(x)$ and $F_15(x)$ in the first column,



In the righthand column F_1, F_3 and F_{11} for the example on the previous page are shown.