Class 33: Monday November 25

Applying Convergence Tests to Infinite Series; Intro to Fourier Series

We now have a stable of convergence tests at our disposal:

Zero-Limit Divergence Test

Absolute Ratio Test

Root Test

Alternating Series Test

Integral Test

Comparison Test

For each of the above tests write down a sentence or mathematical list of symbols which indicate your understanding of how they work. You may also want to indicate which ones you find are more useful than others.

Fourier Series
Taylor used polynomials to approximate functions.
Fourier used trigonometric functions to approximate **periodic** functions.

We write $P_n(x)$ for the nth degree Taylor polynomial.
Example: $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$.

We write $F_n(x)$ for the nth degree Fourier "polynomial."
Example: $F_3(x) = a_0 + a_1\cos(x) + a_2\cos(2x) + a_3\cos(3x) + b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x)$.
a_k and b_k are some constants. They are called the coefficients.

A Taylor Series is $\sum_{k=1}^{\infty} a_k x^k$. A Fourier Series is $a_0 + \sum_{k=1}^{\infty} b_k \sin(kx) + a_k\cos(kx)$

For a periodic function $f(t)$ whose period is 2π, the coefficients of its Fourier Series are:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)dt \qquad a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(kt)dt,\ k = 1, 2, 3, \cdots$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(kt)dt,\ k = 1, 2, 3, \cdots$$

EXAMPLE

$$f(x) = \begin{cases} 7 & \text{if } (2n)\pi \le x \le (2n+1)\pi \\ 0 & \text{if } (2n+1)\pi < x < (2n+2)\pi \end{cases}$$

1. Sketch the graph of $f(x)$ below.

2. Find the first degree Fourier polynomial for $f(x)$.

3. Find the second degree Fourier polynomial for $f(x)$.