Recall that we can use The Microscope Equation to approximate any differentiable function \( f(x) \) near the point \( x = a \) using the expression:

\[
\Delta f \approx f'(a) \cdot \Delta x
\]

which is also written as

\[
f(x) \approx f(a) + f'(a)(x - a)
\]

It is this property which allows us to write \( \sin(x) \approx x \) and \( \cos(x) \approx 1 \) near \( x = 0 \). Do you see how you can use the Microscope Equation to get these results?

\[
\sin(x) \approx \sin(0) + \cos(0)(x - 0)
\]

\[
\cos(x) \approx \cos(0) + -\sin(0)(x - 0)
\]

What is the meaning of the Microscope equation, graphically or visually?

Taylor’s Theorem allows us to approximate a continuous, differentiable function \( f(x) \) by a polynomial, and not just by a line. The definition of a Taylor Polynomial of degree \( n \) for \( f(x) \) about a point \( x = a \) is given by:

\[
P_n(x; a) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2 + \frac{f'''(a)}{6}(x - a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x - a)^n = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x - a)^k
\]

**Examples**

Find the Taylor Polynomial of degree 7 for \( f(x) = \cos(x) \) and \( g(x) = \sin(x) \) about \( x = 0 \). Note what information you need to find a Taylor Polynomial:

1. \( \sin(x) \approx \)  

2. \( \cos(x) \approx \)

Notice any patterns?
Some Taylor polynomials are fun to compute. For example, we can write down the $n$-th degree Taylor Polynomial for $f(x) = e^x$ about $x = 0$ below:

3. $e^x \approx$

Another interesting one is the $n$-th degree Taylor Polynomial for $f(x) = \frac{1}{1-x}$ about $x = 0$

4. $\frac{1}{1-x} \approx$

**Patterns of Taylor Polynomials**

When computing Taylor Polynomials it helps to notice patterns so we don’t have to differentiate our lives away...

Consider the function $\frac{1}{1+x}$. How is it different from 4. above?

What does your intuition tell you it’s Taylor Polynomial will look like...?

5. $\frac{1}{1+x} \approx$

**GroupWork**

Try to write the general Taylor Polynomial (of degree $n$, about $a = 0$) for the following functions by seeing a relationship between the given function and a function you already know the Taylor Polynomial for. (You can always check your answer the hard way....)

6. $\frac{1}{(1-x)^2} \approx$

7. $e^{ax^2} \approx$

8. $e^{5x} \approx$

9. $\ln(1+x) \approx$