Class 26: Wednesday November 6

Nonlinear Oscillator Models

We previously derived the initial value problem for the motion of a spring.

$$\begin{array}{ll} x' = v & x(0) = a \\ v' = -b^2 x & v(0) = p \end{array}$$

Recall $b^2 = c/m$. We can also think of the IVP as a second order differential equation for $x(t)$.

$$x'' = -b^2 x, \qquad x(0) = a, \qquad x'(0) = 0$$

The Hard Spring

In our derivation of the linear spring model we assumed that the displacement x of the spring was directly proportional to the applied force, to obtain the equation $F = -kx$.

In the non-linear hard spring model, we can have a situation where to in order to double the displacement you have to MORE than double the force, i.e. you have a hard spring, where now the relationship between Force applied and displacement is $F = -cx - \gamma x^3$, $c > 0$, $\gamma > 0$

Sketch the Force versus Displacement graph for the hard spring in the space below:

we can show that the corresponding IVP will be:

$$\begin{array}{ll} x' = v & x(0) = a \\ v' = -b^2 x - \beta x^3 & v(0) = p \end{array}$$

What is the behavior of the hard spring like for SMALL displacements, i.e. x much less than 1?

The Pendulum

In the pendulum problem, the relationship between Force and Angular Displacement is $F = -mg\sin(x)$.

Sketch the Force versus Displacement graph for the pendulum model in the space below:

The corresponding IVP for the pendulum model will be:

$$\begin{array}{ll} x' = v & x(0) = a \\ v' = -g\sin(x) & v(0) = p \end{array}$$

The pendulum can be considered to behave like a soft spring. Why?

Again, what is the behavior of the pendulum similar to for small values of x?

If you wanted to graph solutions of these models what method would you use?