Introduction to Periodic Functions

There are many functions and systems which exhibit periodic behavior. The textbook refers to the sine and cosine functions as the circular functions since one can define all the points on a unit circle centered at the origin using the equations

\[ x = \cos(t), \quad y = \sin(t) \]

where \( t \) is a measurement of the angular displacement from the positive \( x \)-axis.

The sine and cosine functions are also the most basic examples of periodic functions. A function \( f \) is periodic with a period \( T > 0 \) if and only if

\[ f(t + T) = f(t). \]

1. Is \( 2\pi \) a period of \( \cos(t) \)? Is \( 4\pi \) a period of \( \cos(t) \)? Is \( 6\pi \) a period of \( \cos(t) \)? What is the smallest period of \( f(t) = \cos(t) \)?

The smallest period of \( f \) is said to be the period of \( f \). The frequency of \( f \) is defined to be \( 1/T \), where \( T \) is the period of \( f \).

2. Find the period and frequency of \( f(t) = \cos(t) \).

3. Find the period and frequency of \( g(t) = \cos(2t) \).

4. The amplitude of a periodic function is the largest value the function attains, regardless of sign. Find the period, frequency and amplitude of \( h(t) = \cos(nt) \), where \( n \) is a positive integer.
5. Consider \( f(x) = x^2 \) and \( g(x) = f(x + 2) \) and \( h(x) = f(x - 1) \). Sketch \( f(x) \), \( g(x) \) and \( h(x) \) below (on the same axes).

(a) \( g(x) = f(x + 2) \) is the same function as \( f(x) \) SHIFTED ________ UNITS TO THE ________

(b) \( h(x) = f(x - 1) \) is the same function as \( f(x) \) SHIFTED ________ UNITS TO THE ________

6. Look at the graphs of \( \sin(x) \) and \( \cos(x) \) below and then answer the questions below.

(a) \( \sin(x) \) is the same function as \( \cos(x) \) SHIFTED ________ UNITS TO THE RIGHT.

(b) \( \cos(x) \) is the same function as \( \sin(x) \) SHIFTED ________ UNITS TO THE RIGHT.

(c) Therefore we could write \( \sin(x) = \cos(x - \phi) \). What is the value of \( \phi \)?

(d) We can also write \( \cos(x) = \sin(x - \phi) \). What is the value of \( \phi \)?

(e) Is there more than one value of \( \phi \)? Is there more than one value of \( \alpha \)? These numbers are called phase shifts.

7. A function is said to be even if \( f(-x) = f(x) \). It is said to be odd if \( f(-x) = -f(x) \). A nonzero function can either be classified as even, odd or neither.

(a) Is \( \sin(x) \) even, odd or neither?

(b) Is \( \cos(x) \) even, odd or neither?