Class 23: Wednesday October 23

Introduction to Periodic Functions

There are many functions and systems which exhibit periodic behavior. The textbook refers to the sine and cosine functions as the *circular functions* since one can define all the points on a unit circle centered at the origin using the equations

$$x = \cos(t), \qquad y = \sin(t)$$

where t is a measurement of the angular displacement from the positive x-axis.
The sine and cosine functions are also the most basic examples of *periodic functions*. A function f is **periodic** with a period $T > 0$ if and only if

$$f(t + T) = f(t).$$

1. Is 2π a period of $\cos(t)$? Is 4π a period of $\cos(t)$? Is 6π a period of $\cos(t)$? What is the *smallest* period of $f(t) = \cos(t)$?

   The *smallest* period of f is said to be **the period** of f. The **frequency** of f is defined to be $1/T$, where T is the period of f.

2. Find the period and frequency of $f(t) = \cos(t)$.

3. Find the period and frequency of $g(t) = \cos(2t)$.

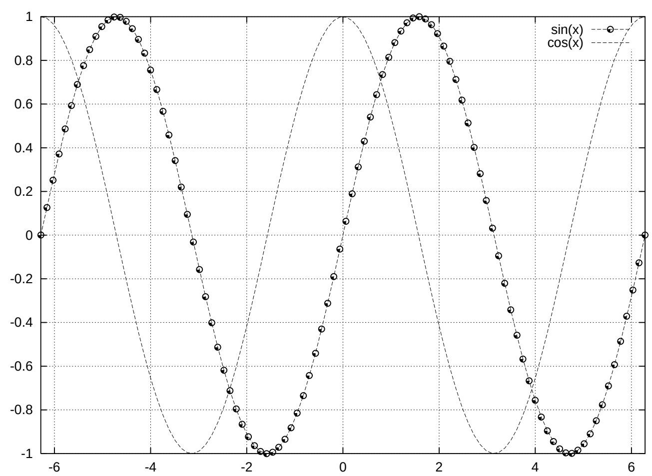
4. The **amplitude** of a periodic function is the largest value the function attains, regardless of sign. Find the period, frequency and amplitude of $h(t) = \cos(nt)$, where n is a positive integer.

5. Consider $f(x) = x^2$ and $g(x) = f(x+2)$ and $h(x) = f(x-1)$. Sketch $f(x)$, $g(x)$ and $h(x)$ below (on the same axes).

(a) $g(x) = f(x+2)$ is the same function as $f(x)$ SHIFTED __________ UNITS TO THE _______________

(b) $h(x) = f(x-1)$ is the same function as $f(x)$ SHIFTED __________ UNITS TO THE _______________

6. Look at the graphs of $\sin(x)$ and $\cos(x)$ below and then answer the questions below



(a) $\sin(x)$ is the same function as $\cos(x)$ SHIFTED __________ UNITS TO THE RIGHT.

(b) $\cos(x)$ is the same function as $\sin(x)$ SHIFTED __________ UNITS TO THE RIGHT.

(c) Therefore we could write $\sin(x) = \cos(x - \phi)$. What is the value of ϕ?

(d) We can also write $\cos(x) = \sin(x - \phi)$. What is the value of ϕ?

(e) Is there more than one value of ϕ? Is there more than one value of α? These numbers are called **phase shifts**.

7. A function is said to be **even** if $f(-x) = f(x)$. It is said to be **odd** if $f(-x) = -f(x)$. A nonzero function can either be classified as even, odd or neither.

(a) Is $\sin(x)$ **even**, **odd** or **neither**?

(b) Is $\cos(x)$ **even**, **odd** or **neither**?