Class 20: Wednesday October 23

Definition of the Definite Integral

Area as a definite integral

What is the area under the curve $f(x) = 3x$ from the origin ($x = 0$) to some point $x = L$? Sketch a picture of this shape in the space below and compute its area. (We can do this without any Calculus.)

Write down a Riemann Sum using an equipartition on [0,L] with N subintervals to approximate the area. In order to do this you will need to answer a few questions:

What is Δx for your partition? (What is the *general* formula for Δx for an equipartition from a to b?)

What is the formula for the sampling point x_k? (How does the sampling point change if we use Left Riemann Sums, Right Riemann Sums or Midpoint Riemann Sums?)

What is a formula for the **exact area** under the curve, using the Right Hand Riemann Sum? (How do estimates made using Riemann Sums become more accurate?)

You should obtain an expression which involves L, N and $\sum_{k=1}^{N} k$

Using the fact that $\sum_{k=1}^{N} k = \frac{N}{2}(N+1)$ we can obtain an expression for our Right Hand Sum approximation which involves L and N only.

To make our Riemann Sum approximation be exactly equal to the area we are computing what do we have to do to N? Do it!

We can represent the exact value of the area A by the symbol $\int_0^L 3x\,dx$. This symbol is called a **definite integral**. Is the value of this definite integral the same as the number you obtain from the Riemann Sum?

Notice if we had used Left Hand or Midpoint Riemann Sums we would still get the same answer in the end.

Definition of the Definite Integral

$$\lim_{N\to\infty} \sum_{k=1}^{N} f(x_k)\Delta x_k = \int_a^b f(x)\,dx$$

when $f(x)$ is some function defined on an interval $[a, b]$ split into N subintervals consisting of the slices Δx_k