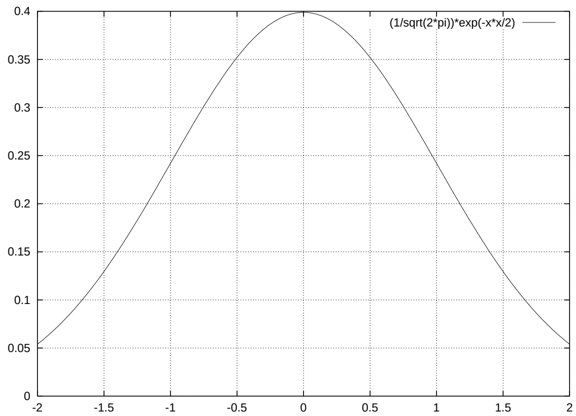
Class 18: Friday October 18

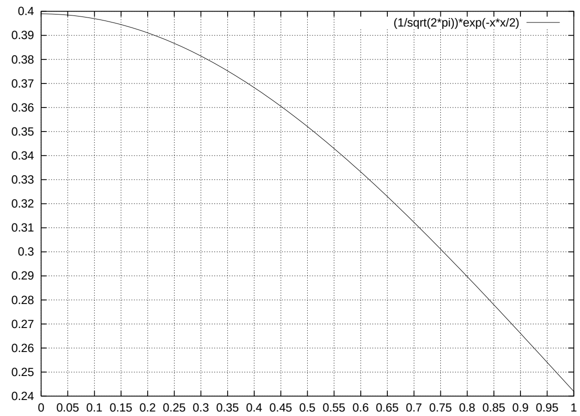
Numerical Methods for Approximating Definite Integrals

The Normal Density Function.

In Japan, trains run on a very precise schedule and tend to arrive in the station, if not exactly on time, very close to the scheduled arrival time.

Suppose a train, the Nagoya Express, is scheduled to arrive daily in Tokyo at 12 noon. The train company keeps records on the actual arrival times of the train each day and finds that the graph of the relative frequency of arrival times tends to look like a "bell curve" that is centered at 12 noon (which we will set to be time $t = 0$ for convenience).





A model for the relative frequency is the *normal density function*

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}.$$

On a graphing calculator, we can plot this function with $-2 \leq t \leq 2$ and $0 \leq f(t) \leq .40$ to check that it does indeed look like a bell curve (see figures above).

We will use this density function to find the probability that the Nagoya Express arrives at the station between 12 noon and 12:01. This is equivalent to finding **the area under $f(t)$ between $t = 0$ and $t = 1$.** (In probability lingo, we are finding the probability that the arrival time is within one standard deviation above the mean, where the "mean" is the expected arrival time, i.e., noon, and the "standard deviation" is a measure of the spread of the arrival times about the mean.)

Mathematically, this involves us computing the value of $I = \int_0^1 \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ exactly. Can you do this?

We can NOT always obtain the EXACT value of a definite integral, but we can ALWAYS approximate the value of an integral using Riemann Sums and other numerical integration techniques. In fact, if we have enough time and computing power, we can theoretically obtain a value of the definite integral which is as accurate as we want it to be.

Let's try to get an approximation for $I = \int_0^1 \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ by filling in some of the values in the table.

# of Subintervals	Left Endpoint	Right Endpoint	Midpoint	Trapezoid	Simpson's
2					
4					
$\vdots$					
1000	0.341423212	0.341266240	0.341344726	0.341344756	0.341344746

Analysis of Results

1. Determine whether the density function

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}.$$

is increasing or decreasing on the interval $[0, 1]$.

2. Determine the concavity of the density function $f(t)$ on the interval $[0, 1]$.

3. Which numerical methods produce over-estimates of I? Which produce under-estimates? Can you explain why? (HINT: What do your calculations in (1) and (2) have to do with your answer?)