*Class 5:* Friday September 13

**More fun with the S-I-R Model**

Given

$$\begin{aligned} S' &= -0.00001SI \\ I' &= 0.00001SI - (2/25)I \\ R' &= (2/25)I \\ S_0 = 35000 \qquad I_0 &= 100 \qquad R_0 = 4900 \end{aligned}$$

GROUPWORK:

**1.** During class today, we will try to develop IVPs for the following situations which modify the given S-I-R model above.

a. **Vaccination.** A modification of the original SIR model above after a partially successful vaccine is given to the population which cuts the infectiousness down to one quarter of its present infectiousness.

b. **Improve treatment.** A modification of the original SIR model after a treatment is discovered which reduces the time one is sick to 3 days.

c. **Immunity Loss.** A modification of the original SIR model so that 1 out of every 200 persons who recover become susceptible again.

d. **Death.** A modification of the initial SIR model so that 1 out of every 30 persons who are infected dies, the rest recover.

**Asymptotic Analysis**

**2.** Consider the generic S-I-R model with parameters $a$, $b$ and $c$. What happens after the epidemic has been running through the population for a very very long time? In other words, what happens as $t \to \infty$? What is $S_\infty = \lim_{t\to\infty} S$? $I_\infty = \lim_{t\to\infty} I$? $R_\infty = \lim_{t\to\infty} R$?

a. Show that $(I + S - (b/a) \cdot \ln S)' = 0$

b. Therefore show that $S_\infty - \frac{b}{a}\ln(S_\infty) = I_0 + S_0 - \frac{b}{a}\ln(S_0)$

c. What do these results above tell us about the possibility of someone never getting the disease? In other words, what is the value of $S_\infty$?