Class 3 Monday September 9

Initial Value Problems and Euler's Method

An **Initial Value Problem** consists of an ordinary differential equation coupled with an initial condition. For example, our population model is expressed as an IVP:

$$P' = 0.017P, \qquad P(0) = 100$$

What's the population at $t = 1$? What about at $t = 2$? $t = 10$? How would we improve these estimates?

Euler's Method

There is a standard technique for approximating the solution of initial value problems, called *Euler's Method* (pronounced "Oilers Method"). It is named after Leonhard Euler (1707-1783), a great Swiss mathematician who contributed extensively to the development of the Calculus. It is based on the interpretation of the derivative as a slope, and on the *Microscope Approximation* for a differentiable function $y(t)$: $\Delta y \approx y'(t)\Delta t$

Mathematically, given the IVP

$$y' = f(t, y), \qquad y(a) = b$$

we can express Euler's Method as:

$$\begin{aligned} y_{new} &= y_{old} + \Delta y \\ \Delta y &= y' \cdot \Delta t \end{aligned}$$

OR, combining the two into one step gives us

$$y_n \approx y_{n-1} + y'_{n-1} \cdot \Delta t$$

Exercise

Consider the IVP $y' = t \cdot y, \quad y(0) = 2$. Compute **four steps** of Euler's Method to approximate the solution at $y(4)$. Fill out the table below.

t	y	y'	Δy	Δt