NAMES: ______________________________

Lab 7: Investigating Trigonometric Functions

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1. Preparation

Read 7.1 and 7.2 in *Calculus in Context*. Our goal is to get a sense of what weighted linear combinations of sine and cosine look like.

Fill in the blanks in the questions on this page and then you will use these identities to assist you with completing the rest of the lab worksheet.

(a) $\sin(-x) =$ ____________

(b) $\cos(-x) =$ ____________

(c) Find $\phi \in [0, \pi]$ such that $\cos(x - \phi) = \sin(x)$.

(d) Find $\phi \in [0, \pi]$ such that $\sin(x + \phi) = \cos(x)$.

Note: The identities in (c) and (d) allow every sine expression to be converted into a cosine expression and every cosine expression to be converted into a sine expression.

(e) Recall the identity

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B).$$

Using the ϕ value you found in part (c) above, we can write

$$\sin(A + B) = \cos\Big((A + B) - \phi\Big) = \cos\Big(A + (B - \phi)\Big).$$

Use the cosine of a sum identity and the results of parts 1(c) and 1(d) to deduce the identity for the sine of a sum.

$$\sin(A + B) \;=\; \cos\Big(A + (B - \phi)\Big) \;=$$

2. First Exercise.

For this problem, let $f(x) = \cos x + \sin x$.

(a) Show that 2π is **a** period of the function f (use the definition of periodic and the identities from 1(e)).

(b) Use the derivative $f'(x)$ to determine the maximum and minimum values of f over the interval $[0, 2\pi]$.

(c) Use both of the previous results to **explain** why 2π is **the** period of the function f. [HINT: how many times does f attain its maximum and minimum values you obtained in part (b)?]

(d) Use the sum identity for cosine to verify that $\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = \cos x + \sin x$.

(e) On separate paper, sketch the graph of the function f, and check your answer to part 2(b). Note that you now have two ways of looking at this function (from part 2(d)). Choose your scales carefully and accurately. (Attach the graph to this lab.)

3. Second Exercise

For this problem, let $h(x) = \cos x + \sqrt{3}\sin x$.

(a) Show that 2π is **a** period of the function h. (Again, use the definition of periodic and the identities from 1(e).)

(b) Determine the maximum and minimum values of the function h on $[0, 2\pi)$.

(c) What is **the** period of the function h? How do you know?

(d) On separate paper, sketch the graph of the function h carefully and accurately. Check your graph with a calculator or with Derive on a computer. (Attach your graph to this lab.)

(e) Using your knowledge about the max/min of h, determine the numbers C and ϕ so that $h(x) = C\cos(x - \phi)$. Explain your reasoning.

4. Generalization

Here, we consider a general combination of a sine and cosine with the same period. For this problem, let A and B denote *positive* numbers.

(a) If $\tan\phi = \dfrac{B}{A}$, verify that $\cos\phi = \dfrac{A}{\sqrt{A^2 + B^2}}$. (Hint: Either set up a triangle with angle ϕ or draw the point ϕ radians around the unit circle.)

(b) What is the analogous formula for $\sin\phi$?

(c) For positive A and B, let

$$S(x) = A\cos\omega x + B\sin\omega x \qquad \text{and} \qquad D(x) = A\cos\omega x - B\sin\omega x.$$

Let $C = \sqrt{A^2 + B^2}$ and $\phi = \arctan(\frac{B}{A})$. Use the identity from 1(e) and the information developed above to verify that

$$S(x) = C\cos(\omega x - \phi) \qquad \text{and} \qquad D(x) = C\cos(\omega x + \phi).$$

(d) What are the period, frequency, amplitude, and phase shift of the functions S and D?

	period	frequency	amplitude	phase shift
S				
D				

(e) Find S' and S''. Express S'' in terms of S. (You'll see this again in class...)

5. Third Exercise

Now let $j(x) = \sqrt{3}\cos 3x + 2\sin 3x$. Determine the numbers C and ϕ so that $j(x) = C\cos(3x - \phi)$. Sketch the graph on separate paper, and attach it to this lab.

6. Assignment

Each team must turn in one **neat** copy of this worksheet, completely filled out You should attach three sheets of paper which contain plots of $f(x)$, $h(x)$ and $j(x)$. On these sheets you should also write down the second-order equation that each functions solves, i.e. $S'' + \omega^2 S = 0$.

Each team must turn in one **neat** copy of this worksheet, completely filled out on **Thursday November 7**.