NAMES: ______________________________

Lab 6: Techniques of Antidifferentiation

Preparation.

Today we will go over the derivation of some basic antiderivatives. Then we will practice these techniques and check our work with Derive. This is to assist you in passing the Integration Gateway test.

To warm up, work with your group to develop a "top ten" list of antiderivatives to know. Prepare your list on one of the blank table forms.

The Scaling Principle.

Suppose that we know that the function g is an antiderivative of the function f. Then we have two notations to present this information:

$$\frac{d}{dt}\,g(t) = f(t) \qquad \text{or} \qquad \int f(t)\,dt = g(t)$$

If we change the scale of the input variable by any nonzero scaling factor a, then we can express the rate of change of g by using the chain rule.

$$\frac{d}{dx}\,g(ax) = \underline{\hspace{3cm}}$$

(This expresses the principle that permits you to convert a velocity from "mile per hour" to "miles per day" using the conversion factor $a = 24$.)

Using algebra, we can change the information into a conclusion about antiderivatives:

$$\frac{d}{dx}\Big[\underline{\hspace{3cm}}\Big] = f(ax) \qquad \text{or} \qquad \int f(ax)\,dx = \underline{\hspace{3cm}}$$

This leads us to the *Scaling Principle* as a conclusion:

$$\text{If } \int f(t)\,dt = g(t), \text{ then } \int f(ax)\,dx = \frac{1}{a}\,g(ax).$$

There is a companion scaling principle which can be deduced using the renaming $a = 1/b$:

$$\text{If } \int f(t)\,dt = g(t), \text{ then } \int f\left(\frac{x}{b}\right)\,dx = \left[\rule{2cm}{0.4pt}\right].$$

The Shifting Principle.

When the point of origin (or "frame of reference") is moved, this also affects the form of expressions, but not their essential content. Knowing that

$$\frac{d}{dt}\,g(t) = f(t) \qquad \text{or} \qquad \int f(t)\,dt = g(t),$$

we can conclude that

$$\frac{d}{dt}\,g(t-c) = f(t-c) \qquad \text{or} \qquad \int f(t-c)\,dt = \left[\rule{2cm}{0.4pt}\right].$$

(This can be interpreted by saying that knowledge of the velocity in Pacific time implies the same knowledge using Greenwich Mean Time.)

Rational Functions.

Here we will focus on rational functions with quadratics as denominators.

Recall that

$$\int \frac{1}{x^2+1}\,dx = \arctan x.$$

a. Use the scaling principle to determine a formula for the following antiderivative:

$$\int \frac{1}{x^2+9}\,dx = \rule{2cm}{0.4pt}.$$

b. Use the shifting principle to determine a formula for the following antiderivative:

$$\int \frac{1}{x^2-2x+2}\,dx = \rule{2cm}{0.4pt}.$$

Next we will find a companion antiderivative formula for

$$\int \frac{1}{x^2-1}\,dx.$$

Note first that the denominator can be factored into two linear terms. Use algebra to determine the values of A and B.

$$\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

Now, using the shifting principle, determine formulas for the following antiderivatives:

$$\int \frac{A}{x-1}\,dx = \underline{\hspace{4cm}}.$$

$$\int \frac{B}{x+1}\,dx = \underline{\hspace{4cm}}.$$

Use what you have calculated above to determine a formula for this antiderivative:

$$\int \frac{1}{x^2-1}\,dx = \underline{\hspace{6cm}}.$$

Now calculate the following derivative and explain its connection to the antiderivative above.

$$\frac{d}{dx} \ln \sqrt{\frac{x-1}{x+1}}$$

(For which x-values are these formulas valid?)

Antiderivative of the Log Function.

Calculate the derivative of the product below:

$$\frac{d}{dx}\, x \ln x = \underline{\hspace{4cm}}.$$

Now rearrange terms so that you can fill in the blank below:

$$\frac{d}{dx}\Big[\underline{\hspace{4cm}}\Big] = \ln x.$$

And convert this to an integral form:

$$\int \ln x \, dx = \underline{\hspace{4cm}}.$$

Practice.

Use the techniques developed in this lab to complete the antiderivatives in the table attached. To check your work, go to `Derive` on the computer, `Author` each expression, and then push the $\int$ button. Leave the limits of integration blank and just hit the `Simplify` button.

$f(x)$	$\int f(x)dx$
$\sin x$	
$\sin(ax)$	
$\sin \frac{x}{b}$	
$\frac{1}{1+x^2}$	
$\frac{1}{1+9x^2}$	
$\frac{1}{1+a^2x^2}$	
$\frac{1}{1+(\frac{x}{b})^2}$	
$\frac{1}{b^2+x^2}$	
$\frac{1}{3+x^2}$	

$f(x)$	$\int f(x)dx$
$\dfrac{1}{1-x^2}$	
$\dfrac{1}{1-9x^2}$	
$\dfrac{1}{1-a^2x^2}$	
$\dfrac{1}{b^2-x^2}$	
$\dfrac{1}{x^2-2x+2}$	
$\dfrac{1}{x^2-2x}$	
xe^x	
xe^{x^2}	
x^2e^x	

$f(x)$	$\int f(x)dx$
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

$f(x)$	$\int f(x)dx$
11.	
12.	
13.	
14.	
15.	
16.	
17.	
18.	
19.	
20.	