NAMES: ________________________________

Lab 4: Visualizing Functions of Two Variables

Introduction

Today we will use the software package called `Derive` which specializes in symbolic computation. This essentially means that it solves problems using the rules of algebra, differentiation, antidifferentiation, and other systems of calculation rather than by numerical approximation. We will also use a TruBasic program called `CONTOUR.TRU`.

The purpose of this lab is to explore, graphically and numerically, functions of two variables. We will do this by using several different techniques to gain information about surfaces determined by functions of two variables.

§1 Vertical Slices

An alternative approach is to view the graph in *slices*. In this section of the lab we will consider how to take *vertical* slices. These slices have the advantage of reducing the problem to graphing functions of one variables, which we can plot easily in a 2-D plane.

DEFINITION:

Suppose $f : R^2 \to R,\ \ z = f(x, y)$ is a function of two variables.

The **vertical slice of f along x, holding $y = b$** is the function

$$g(x) = f(x, b), \quad \text{provided } (x, b) \in U.$$

Similarly, the **vertical slice of f along y, holding $x = a$** is the function

$$\phi(y) = f(a, y), \quad \text{provided } (a, y) \in U.$$

Example

Suppose $z = f(x, y) = 2x^2 + 3xy - 4$

The vertical slice of f along x, holding $y = 3$ is

$$z = f(x, 3) = 2x^2 + 3x \cdot 4 - 4 = 2x^2 - 12x - 4.$$

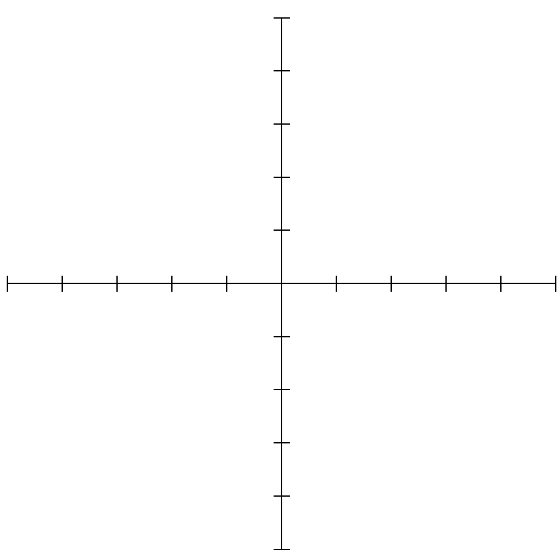
Find the vertical slice of f along x, holding $y = 0$:

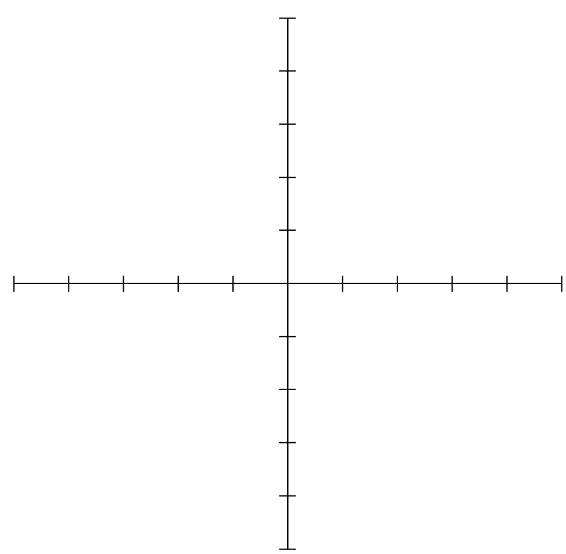
The vertical slice of f along y, holding $x = -1$ is

$$z = f(-1, y) = 2(-1)^2 + 3(-1)y - 4 = -3y - 2.$$

Find the vertical slice of f along y, holding $x = 5/2$:

What do each of these vertical slices look like for this function $f(x, y) = 2x^2 + 3xy - 4$?



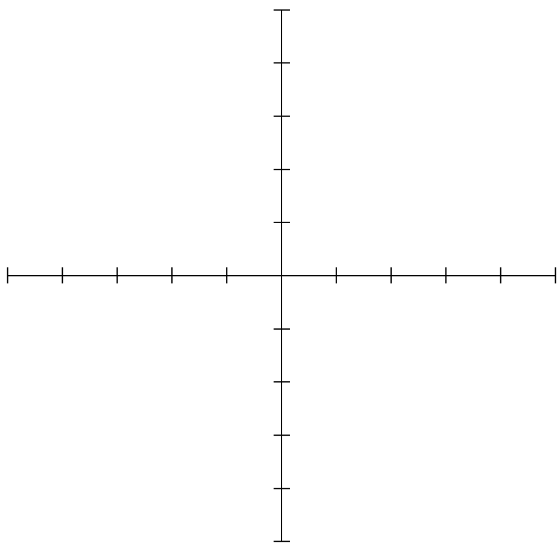


Sketch them on the (above) axes here.

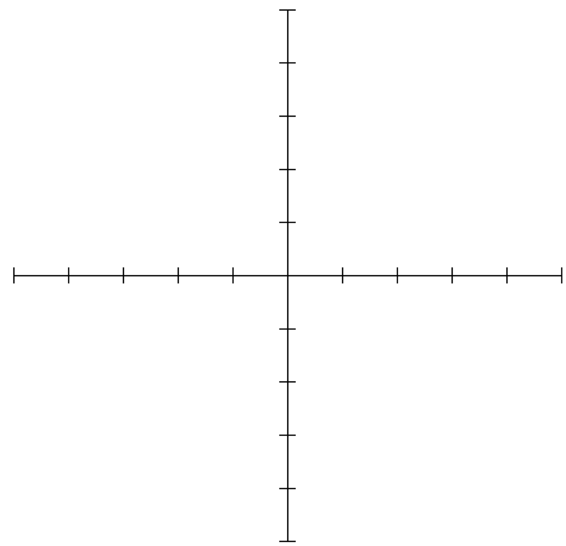
VERTICAL SLICES

$$g(x, y) = x^2 + y^2$$

Along x-axis

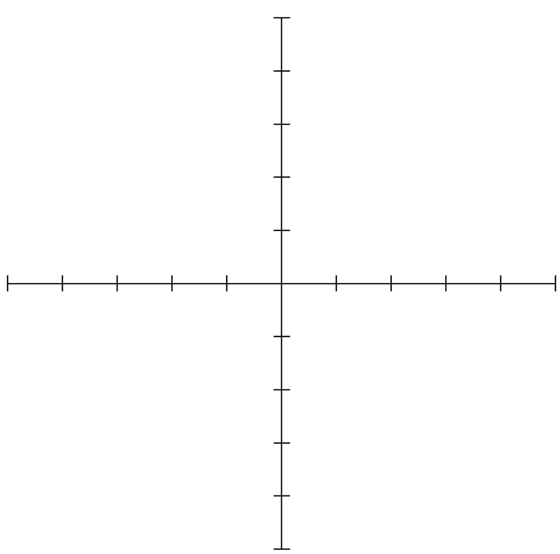


Along y-axis

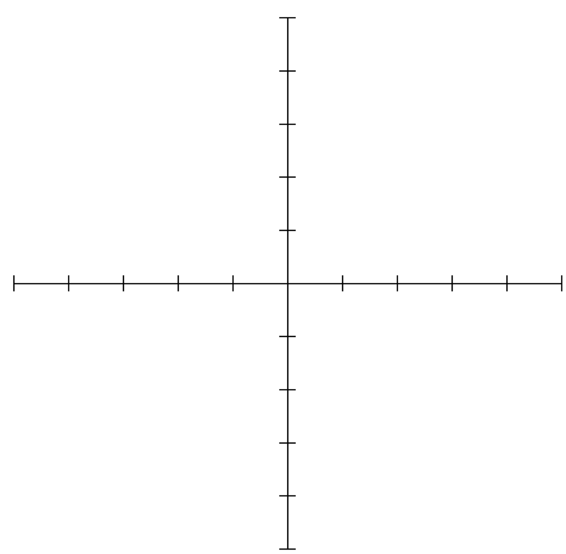


$$j(x, y) = x^2 - y^2$$

Along x-axis



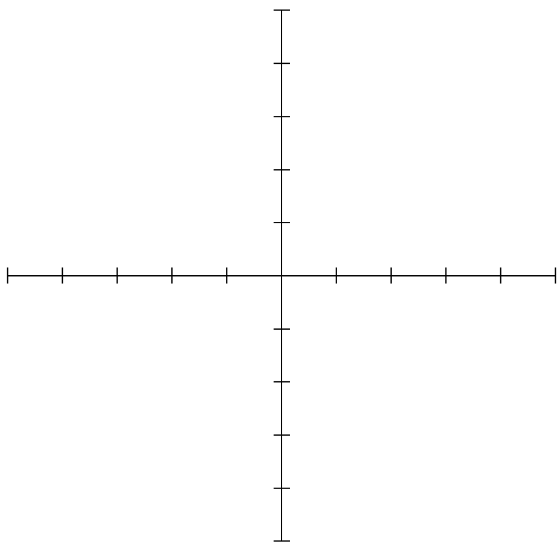
Along y-axis



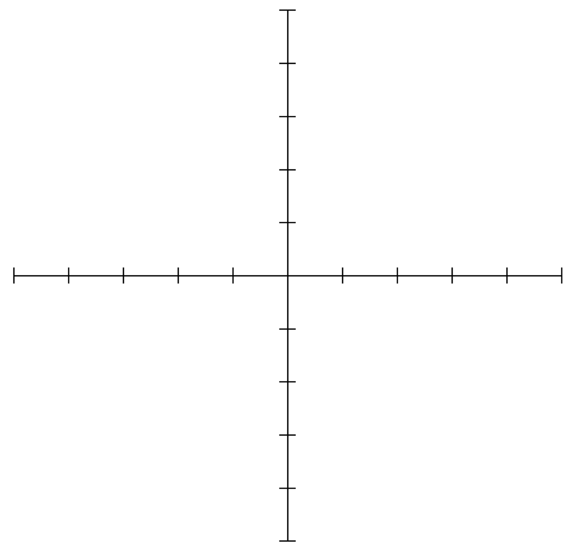
VERTICAL SLICES

$$j(x, y) = (x + y)^2$$

Along x-axis

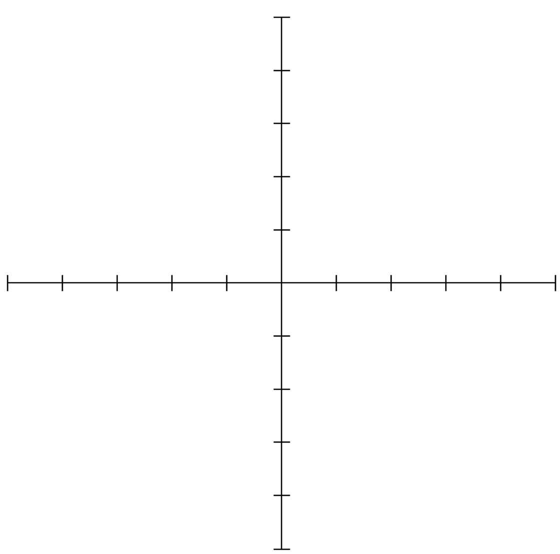


Along y-axis

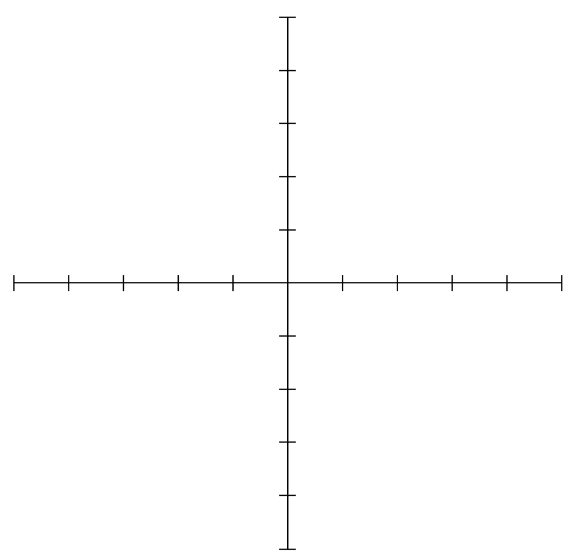


$$k(x, y) = \sin(x) - \sin(y)$$

Along x-axis



Along y-axis



§2 Horizontal Slices

It requires mathematical perception and artistic ability to sketch graphs of functions of two variables. Another alternative to vertical slices is to plot the level curves (or contour lines) for the function on a two dimensional graph .

DEFINITION:

Suppose $f : R^2 \rightarrow R,\ \ z = f(x, y)$ is a function of two variables.

A **contour (or level set) of** f is the set of points for which the function is constant, i.e. $z = f(x, y) = C$. One can almost think of a contour as a **horizontal** slice of the surface $z = f(x, y)$ at the value $z = C$. Note, contours can also be graphed in a 2-D plane.

Let us examine $g(x, y) = x^2 + y^2$. To find the level curve for $C = 1$, we set $g(x, y)$ equal to 1 and plot the points (x, y) which satisfy the equation

$$x^2 + y^2 = 1.$$

If we do the same for $C = 0$ and $C = 4$ and 9, we can get a good idea of the general shape of the three-dimensional graph. Graph your results on the SURFACE PLOTS page. Graph your contours on the CONTOUR PLOTS page. Then do the same for:

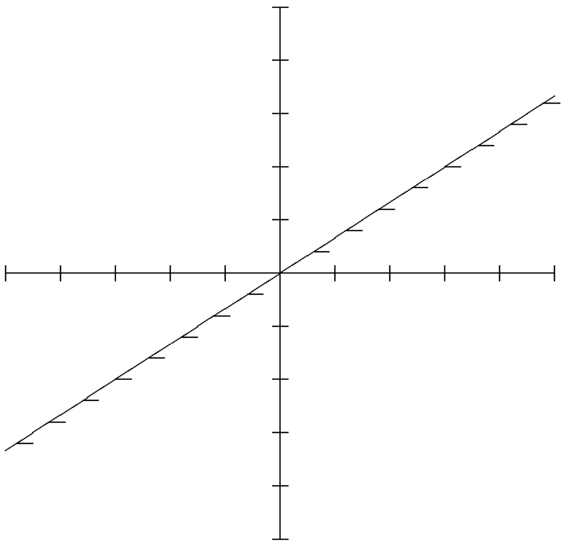
a. $j(x, y)$; $C = -4, -2, 0, 2, 4$

b. $f(x, y)$; $C = 0, 1, 4, 9$

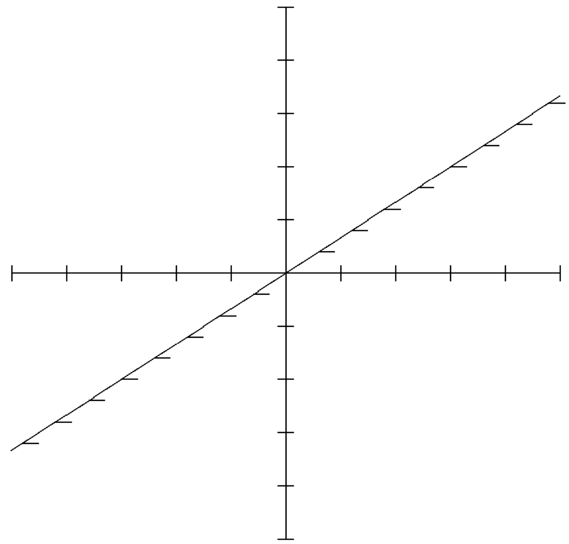
c. $k(x, y)$; $C = -2, 0, 2$

SURFACE PLOTS

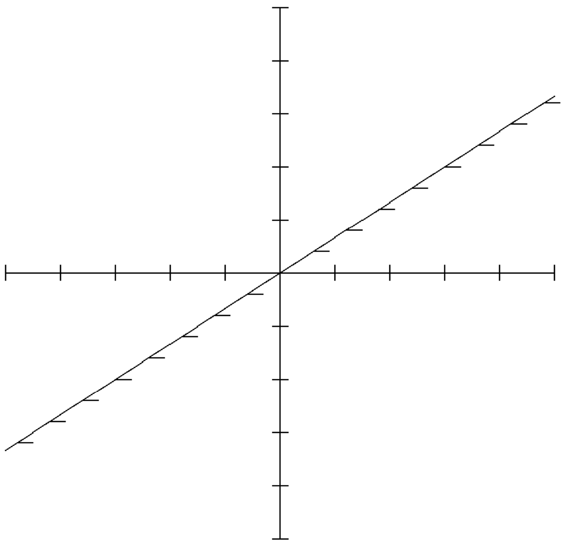
$g(x, y) = x^2 + y^2$



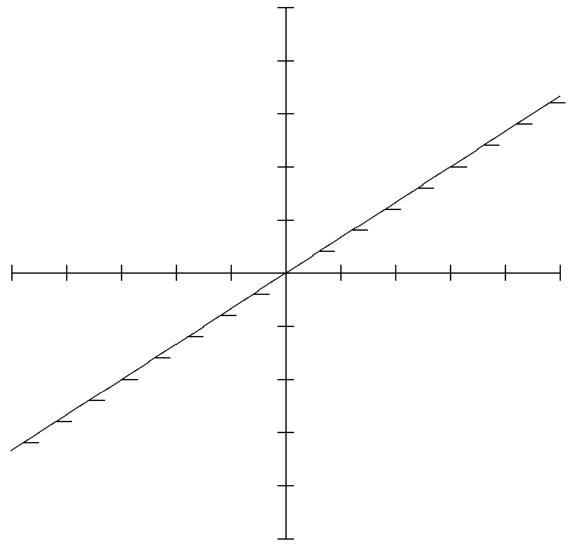
$j(x, y) = x^2 - y^2$



$f(x, y) = (x + y)^2$

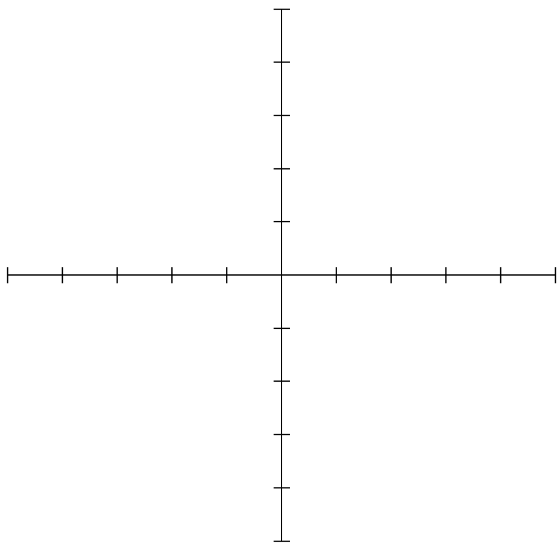


$k(x, y) = \sin(x) - \sin(y)$

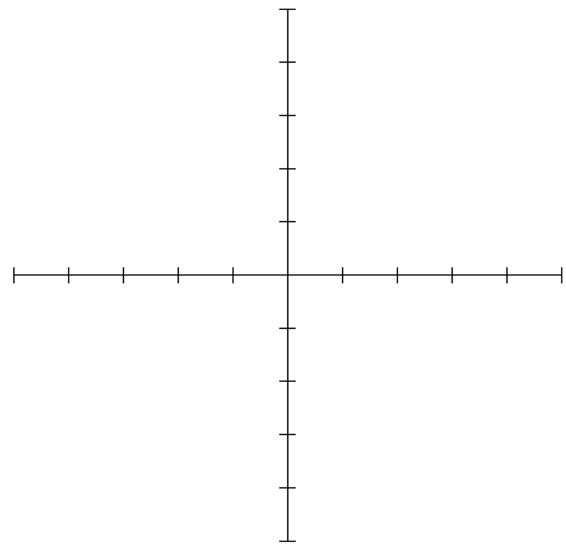


CONTOUR PLOTS

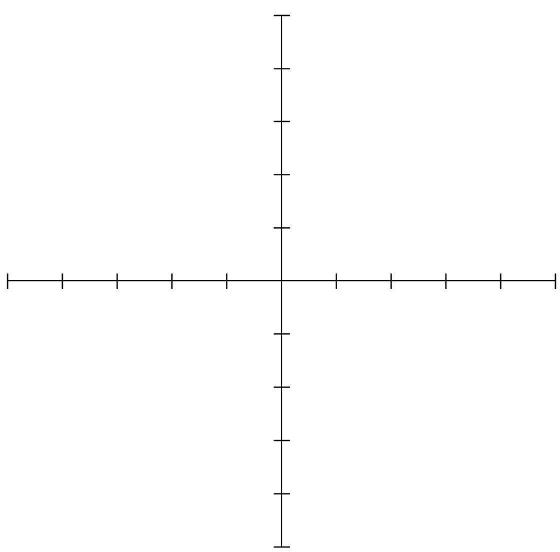
$g(x, y) = x^2 + y^2$



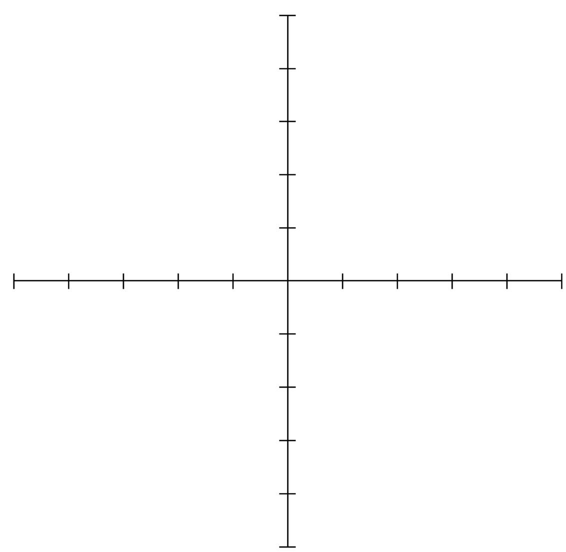
$j(x, y) = x^2 - y^2$



$f(x, y) = (x + y)^2$

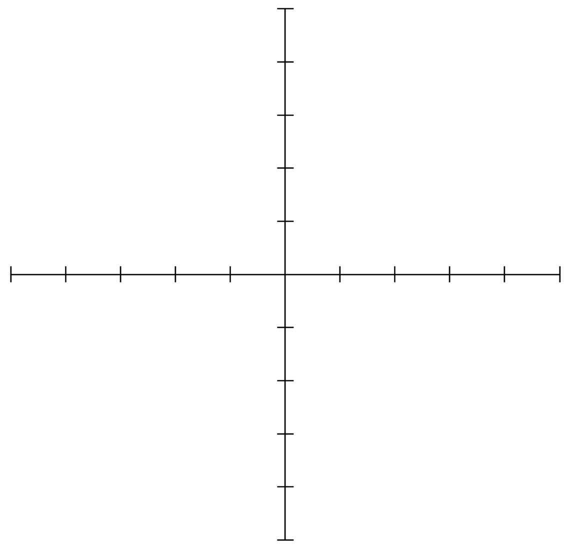


$k(x, y) = \sin(x) - \sin(y)$



Finding the level curves for $k(x, y)$ for constants other than the ones given is a bit tricky. We can, however, use program `CONTOUR.TRU` to graph the contour maps for most functions of two variables.

Open the program `CONTOUR.TRU` and **read through it**. Your task is to set the domain for $k(x, y)$ so that the contour map will include the level curves you have already computed. (Don't forget to type in the definition of $k(x, y)$!). Sketch the result below.



§3 Partial Derivatives

Compute the partial derivatives for all of the functions discussed in this lab and determine the set of points (x,y) for which

(a) the partial with respect to x is 0

(b) the partial with respect to y is 0 and

(c) both partials are 0.

Also, *describe* what is happening geometrically at each point in column [c], i.e. where both partial derivatives are simultaneously zero.

function	[a]	[b]	[c]	geometric description
$g(x, y)$				
$j(x, y)$				
$f(x, y)$				
$k(x, y)$				

Using DERIVE

DERIVE is able to compute partial derivatives by using **C**alculus **D**ifferentiate and choosing either x or y at the appropriate time. You may want to use DERIVE to check your work.

DERIVE can be used to give you a wireframe picture of the graph of a function in two variables. Use the author command to enter your function. (Don't forget the :=) Then click on the 3-d graphics button. Tile your windows, and then click graphics window and on the 3-d button.

§4 Assignment

Write-up: Each team must turn in one copy of this worksheet, completely filled out. Your sketches should approach (in the limit) works of art. In addition, draw the contour map, draw the vertical slices along the x and y axes, find and interpret the partial derivatives, and sketch the graph of the function given to your team, and hand in **Thursday October 17**.