NAMES: ________________________________

Lab 3: Investigating the Predator-Prey Model

Introduction

An Initial Value Problem often contains information about its solutions which one can find without necessarily knowing what the solutions are explicitly. Sometimes a qualitative analysis tells us something about the dynamics of the situation the IVP models. In this lab, we practice extracting information from an IVP using Euler's method and some symbolic manipulations.

It is assumed that you have read pages 163-166 in CiC to familiarize yourself with the Lotka-Volterra predator-prey population model with bounded growth.

§1 The Predator-Prey model and equilibrium

Open the file `PredPrey.xls`. In it you will find three sheets, labeled (1) `Pred-Prey IVP and Euler's Table`, (2) `Time versus Predator and Prey` and (3) `Prey versus Predator`. On the sheet `Pred-Prey IVP and Euler's Table`, create an Euler's table for the IVP given below. (Note that there is an additional column to scale the fox population by 100. This allows us to view the relationship of the two populations more easily on one graph.) Be sure to make the table big enough to show the first three hundred months.

$$\begin{aligned} R' &= aR(1-(R/b)) - cRF \\ F' &= dRF - eF \\ R(0) &= 2000 \\ F(0) &= 10 \end{aligned}$$

with parameters $a = 0.1$; $b = 5000$; $c = 0.005$; $d = 0.00004$; $e = 0.04$. Let $\Delta t = 1$ month.

Look at the graphs on both the sheet labeled `Predator and Prey versus Time` and the sheet labeled `Prey versus Predator` and describe what is occurring as time passes. Typically over time, populations of predator and prey stabilize somewhat. This trend is called finding the natural equilibrium or steady state. You can hone in on the equilibrium graphically by repeatedly replacing the initial values of R and F with the values at the center of the spiral in the graph of `Predator versus Prey`. Do this a few times and describe what happens to the graph `Predator and Prey versus Time`.

Now change the initial values of R and F to $(R(0), F(0)) = (2000,\ 5)$; $(R(0), F(0)) = (2000,\ 30)$; $(R(0), F(0)) = (1000,\ 10)$; $(R(0), F(0)) = (3000,\ 10)$; etc. What differences do you see? What remains the same?

Analytically, the equilibrium occurs when both the rate equations are zero simultaneously. Find the various values for R and F which make both R' and F' zero simultaneously.

§2 **Further analysis of the model**

What happens if, at any point in time, the number of foxes suddenly becomes zero? You may investigate this question using the Euler's method table, but you should answer the question by explaining how the rate equations predict what would happen.

What happens if, at any point in time, the number of rabbits suddenly becomes zero? Again, you may investigate this question using the Euler's method table, but you must answer the question using the rate equations.

§3 **Modifying the Predator-Prey Model**

Modify the Predator-Prey model to produce the **Lotka-Volterra Equations**, which look like:

$$\begin{aligned} R' &= aR - cRF \\ F' &= dRF - eF \\ R(0) &= 2000 \\ F(0) &= 10 \end{aligned}$$

with parameters $a = 0.1$; $c = 0.005$; $d = 0.00004$; $e = 0.04$. Let $\Delta t = 1$ month. Use initial conditions of $F(0) = 10, R(0) = 2000$. Make sure you compute for at least 300 months.

Notice how the Lotka-Volterra equations are a simplification of the two-species predator-prey model.

What are the exact equilibrium values of the Lotka-Volterra model? How do your theoretical values compare to the values you obtain from looking at the graph? (What happens if you change the initial conditions to values very close –within 1, or .1 units –to the equilibrium point? What happens if you choose an initial value which is far–10 or 100 units– away from the theoretical equilibrium point?

How does the F versus R graph for the Lotka-Volterra model differ from the same graph obtained from using the original Predator-Prey model?

There are certain assumptions made about the relationship between the predator population and the prey population that are no longer true in the modified version of that model as represented by the Lotka-Volterra equations. For example, what happens when the rabbit or fox population goes to zero in the Lotka-Volterra model?

§4 **Assignment**

Each team will turn in one typed collaborative lab report that reflects the *entire* team's understanding of the lab. The report is due **Thursday October 17** in lab.

Explanations should be clear, complete, phrased in your own words, and written in good English. There should be only *one* topic per paragraph. Feel free to include any data, tables, or graphs that you need to help with your explanations, but be sure to label them and refer to them in the text of your report (otherwise I won't know to look at them). Your report should be *typed* and no more than four pages in length. Any math symbols, graphs, or tables can be written in neatly by hand. Please refer to the *Comments on Lab Team Writing Assignments* handout for more details and guidance.

Your report should be a cohesive paper including:

> A brief introduction to this predator-prey model, defining all variables. An explanation of how the Lotka-Volterra equations are related to the original, but also how the two models differ in their assumptions, which is reflected in the form of the different IVPs.
>
> A description of the equilibrium for each model and a full explanation of how you found each.
>
> An explanation of what happens to the model when either population suddenly becomes zero, and why this fits with the actual populations being modelled.

Make sure you give yourself and the members of your team enough time to produce a draft which can be evaluated and modified in advance of the due date.