Math 118 – Homework #19 SOLUTION (4 points)

CiC, p. 551, # 22, #23abcde

CiC, p. 551, # 22.

Taylor's theorem is not wrong. In the polynomial approximation of $\sin x$ centered at $x = 0$, only the odd degree terms have nonzero coefficients. Note that

$$\lim_{x\to 0} \frac{\sin x - (x - \frac{x^3}{6})}{x^4} = 0$$

and

$$\lim_{x\to 0} \frac{\sin x - (x - \frac{x^3}{6})}{x^5} = .008333\ldots$$

(see page 539). So both ratios have constant limits. The remainder function $R(x) = O(x^4)$ but the coefficient of the x^4 term is 0 and the coefficient of the x^5 term is $.008333\ldots$.

CiC, p. 551, # 23.

a. $\displaystyle \lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{x - \frac{x^3}{6} + O(x^5)}{x} = \lim_{x\to 0} 1 - \frac{x^2}{6} + O(x^4) = 1.$

b. $\displaystyle \lim_{x\to 0} \frac{e^x - (1+x)}{x^2} = \lim_{x\to 0} \frac{(1 + x + \frac{x^2}{2} + O(x^3)) - (1+x)}{x^2} = \lim_{x\to 0} \frac{\frac{x^2}{2} + O(x^3)}{x^2} = \lim_{x\to 0} \frac{1}{2} + O(x) = \frac{1}{2}.$

c. $\displaystyle \lim_{x\to 1} \frac{\ln(x)}{x-1} = \lim_{x\to 1} \frac{(x-1) + O((x-1)^2)}{x-1} = \lim_{x\to 1} 1 + O(x-1) = 1$

d. $\displaystyle \lim_{x\to 0} \frac{x - \sin(x)}{x^3} = \lim_{x\to 0} \frac{x - (x - \frac{x^3}{3!} + O(x^5))}{x^3} = \lim_{x\to 0} \frac{\frac{x^3}{6} + O(x^5)}{x^3} = \lim_{x\to 0} \frac{1}{6} + O(x^2) = \frac{1}{6}.$

e. $\displaystyle \lim_{x\to 0} \frac{\sin(x^2)}{1 - \cos(x)} = \lim_{x\to 0} \frac{x^2 - \frac{(x^2)^3}{3!} + O(x^{10}))}{1 - (1 - \frac{x^2}{2} + \ldots)} = \lim_{x\to 0} \frac{x^2 - \frac{x^6}{6} + O(x^{10})}{\frac{x^2}{2}} = \lim_{x\to 0} 2 - \frac{x^4}{3} + O(x^8) = \lim_{x\to 0} 2 -$ $O(x^4) = 2.$