Math 118 – Homework # 18 SOLUTION (3 points)

CiC, p. 548, #1, #6, #10

CiC, p. 548, # 1.

a. $\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$, so $\frac{\sin(x)}{x} \approx 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!}$. Thus

$$\begin{aligned} \int \frac{\sin(x)}{x}\,dx &\approx \int 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!}\,dx \\ &= x - \frac{x^3}{3\cdot 3!} + \frac{x^5}{5\cdot 5!} - \frac{x^7}{7\cdot 7!} + C \end{aligned}$$

b. $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{3!}$, so $e^{x^2} \approx 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{3!}$. Thus

$$\begin{aligned} \int e^{x^2}\,dx &\approx \int 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{3!}\,dx \\ &= x + \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + C \end{aligned}$$

c. $\sin(x) \approx x - \frac{x^3}{3!}$, so $\sin(x^2) \approx x^2 - \frac{x^6}{3!}$. Thus

$$\begin{aligned} \int \sin(x^2)\,dx &\approx \int x^2 - \frac{x^6}{3!}\,dx \\ &= \frac{x^3}{3} - \frac{x^7}{42} + C \end{aligned}$$

CiC, p. 548, # 6.

a. Given $f(x) = \sin x$.

$$\sin x \approx \sum_{k=0}^{7} \frac{f^{(k)}(\pi)}{k!}(x-\pi)^k = -(x-\pi) + \frac{(x-\pi)^3}{3!} - \frac{(x-\pi)^5}{5!} + \frac{(x-\pi)^7}{7!}$$

b. Given $g(x) = \cos x$.

$$\cos x \approx \sum_{k=0}^{7} \frac{g^{(k)}(\pi)}{k!}(x-\pi)^k = -1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!} + \frac{(x-\pi)^6}{6!}$$

c. Given $h(x) = \sin(3x)$.

$$\sin(3x) \approx \sum_{k=0}^{7} \frac{h^{(k)}(\pi)}{k!}(x-\pi)^k = -3(x-\pi) + \frac{3^3(x-\pi)^3}{3!} - \frac{3^5(x-\pi)^5}{5!} + \frac{3^7(x-\pi)^7}{7!}$$

CiC, p. 549, # 10.

Given $f(x) = (1+x)^{1/2}$. Then

$$\begin{aligned} f'(x) &= \frac{1}{2}(1+x)^{-1/2} \\ f''(x) &= -\frac{1}{4}(1+x)^{-3/2} \\ f^{(3)}(x) &= \frac{3}{8}(1+x)^{-5/2} \\ f^{(4)}(x) &= -\frac{15}{16}(1+x)^{-7/2} \\ f^{(5)}(x) &= \frac{105}{32}(1+x)^{-9/2} \end{aligned}$$

Thus

$$f(x) \approx \sum_{k=0}^{5} \frac{f^{(k)}(0)}{k!} x^k = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \frac{7x^5}{256}$$

Using the Taylor polynomial to approximate $\sqrt{1.1}$, we can write

$$\begin{aligned} \sqrt{1.1} &= \sqrt{1+0.1} = f(0.1) \\ &\approx \sum_{k=0}^{5} \frac{f^{(k)}(0)}{k!}(0.1)^k \\ &= 1.048808867 \end{aligned}$$

Since $f(0.1) = 1.048808848\ldots$, we see that the estimate with the Taylor polynomial is accurate to 7 decimal places.