Math 118 – Homework #14 SOLUTION (6 points)

pp. 365-7: 1, 4bc,7,8bc,14

CiC, p. 365, # 1.

a. $A(X) = \int_2^X 5\, dx \;=\; 5X - 10$. (graph)

b. Yes, $A'(X) = 5 =$ the slope of the graph of $A(X)$.

CiC, p. 366, # 4. (2 points)

b. $A(X) = \int_0^X \sin(x)\, dx$. By the Fund. Thm. of Calc., $A'(X) = \sin(X)$.

c. $A(X) = \int_0^X \cos(x^2)\, dx$. By the Fund. Thm. of Calc., $A'(X) = \cos(X^2)$.

d. $A(X) = \int_0^X \cos(t^2)\, dt$. By the Fund. Thm. of Calc., $A'(X) = \cos(X^2)$.

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CiC, p. 366, # 7.

Given: $A(X) = \int_0^X x^2 - 4x^3\, dx$.
To find critical points of A, we take its derivative and determine where $A' = 0$ and where A' does not exist.

$$A'(X) = X^2 - 4X^3 = X^2(1 - 4X)$$

so $A' = 0$ when $X = 0$ and when $X = 1/4$. And since A' exists for all X, we have no other critical points. To determine local max/min, we can use the Second Derivative test.

$$A''(X) = 2X - 12X^2$$

Then $A''(1/4) = -1/4 < 0$ which tells us that at $X = 1/4$, A has a local maximum.
However, $A''(0) = 0$ which does not tell us anything. We can note that $A' > 0$ for $-\infty < X < 1/4$ which means that A is increasing throughout this interval. That implies that A has neither a max nor a min at $X = 0$.

CiC, p. 367, # 8.

b. The solution to the IVP is $\int_0^x \sin(t^2)\, dt \;+\; 0$.

c. The solution to the IVP is $\int_0^x \sin(t^2)\, dt \;+\; 5$.

CiC, p. 368, # 14.

The average value $\bar{f}$ of a function $f(x)$ on an interval $[a, b]$ is given by

$$\bar{f} = \frac{1}{b-a} \int_a^b f(x)dx$$

So, the average value of $f(x) = px - x^2$ is $\dfrac{1}{1-0} \displaystyle\int px - x^2 dx = px^2/2 - x^3/3\Big|_0^1 = p/2 - 1/3$

The value of p which makes the average value zero would be $p/2 = 1/3$ or $p = 2/3$.