Math 118 – HW #7 SOLUTION

(4 points)

CiC, p. 106, # 14 and #15. p. 127, # 7. p. 147, #12

CiC, p. 106, # 14.

The function $f(x) = x^{4/5}$ is locally linear at $x = 1$. Proving this rigorously would mean showing the equality of the following:

$$\lim_{\Delta x \to 0} \frac{(1+\Delta x)^{4/5} - 1}{\Delta x} = \lim_{\Delta x \to 0} \frac{(1+\Delta x)^{4/5} - (1-\Delta x)^{4/5}}{2\Delta x} = \lim_{\Delta x \to 0} \frac{1-(1-\Delta x)^{4/5}}{\Delta x}.$$

We can find evidence in support of these equalities by

- zooming in on the graph of f at $x = 1$ and observing that the graph looks more and more linear.
- calculating successive approximations of the limits and showing that they seem to be heading for the same value:

Δx	$\frac{(1+\Delta x)^{4/5} - 1}{\Delta x}$	$\frac{(1+\Delta x)^{4/5} - (1-\Delta x)^{4/5}}{2\Delta x}$	$\frac{1-(1-\Delta x)^{4/5}}{\Delta x}$
0.1	0.7923	0.8003	0.8083
0.01	0.7992	0.8000	0.8008
0.001	0.7999	0.8000	0.8001

CiC, p. 106, # 15.

We are given the function $K(x) = x^{10/9}$.

a. The graph of $K(x)$ on the interval $-1 \leq x \leq 1$ looks a lot like the graph of the absolute value function. So from a quick glance, one might think that this function is not locally linear.

b. Zooming in on $K(x)$ at $x = 0$, we see that the graph is not a sharp point at $x = 0$. The graph smoothly turns around from decreasing to increasing at $x = 0$. The last window in the zooming in process looks like a horizontal line.

c. From the horizontal line appearance of $K(x)$ at $x = 0$ on the zoom window, we can see that yes, $K(x)$ is locally linear at $x = 0$.

CiC, p. 127, # 7.

We are given that distance fallen in feet is related to the time of the fall in seconds by $d = 16t^2$. The duration of the fall is 2.5 ± 0.25 seconds. Therefore the error in time measurement is $\Delta t = 0.25$ seconds. The height of the building is equivalent to the distance fallen which is estimated as $16(2.5)^2 = 100$ feet, with an error of

$$\begin{aligned} \Delta d &\approx d'(2.5) \cdot \Delta t \\ &= (32t)|_{t=2.5} \cdot (0.25) \\ &= (32)(2.5)(0.25) = 20 \text{ feet} \end{aligned}$$

Therefore the height of the building is approximately 100 feet with 20 feet of uncertainty.

CiC, p. 147, # 12.

a. Given that $w = \sqrt{1+x}$, we can determine $w'(x) = \frac{1}{2}(1+x)^{-1/2}$. Therefore the general microscope equation for w is

$$\Delta w \approx \frac{1}{2}(1+x)^{-1/2} \cdot \Delta t.$$

When $x = 0$, we have

$$\Delta w \approx \frac{1}{2}(1+0)^{-1/2} \cdot \Delta t = \frac{1}{2}\Delta t.$$

b. To approximate the value of $\sqrt{1.1056}$, we see that this is the value of $w(0.1056)$. The change in x from 0 to 0.1056 is given by $\Delta x = 0.1056$. Using the microscope equation for w at $x = 0$, we have $\Delta w \approx \frac{1}{2}(0.1056) = 0.0528$. So

$$\sqrt{1.1056} = w(0.1056) = w(0) + \Delta w \approx 1 + 0.0528 = 1.0528.$$

The value of $\sqrt{1.1056}$ to six decimal places of accuracy is 1.051475, so this estimate is accurate to two decimal places.

To approximate the value of $\sqrt{0.9788}$, we see that this is the value of $w(-0.0212)$. The change in x from 0 to -0.0212 is given by $\Delta x = -0.0212$. Using the microscope equation for w at $x = 0$, we have $\Delta w \approx \frac{1}{2}(-0.0212) = -0.0106$. So

$$\sqrt{0.9788} = w(-0.0212) = w(0) + \Delta w \approx 1 - 0.0106 = 0.9894.$$

The value of $\sqrt{0.9788}$ to six decimal places of accuracy is 0.989343, so this estimate is accurate to three decimal places.