Math 118 – Homework #4 SOLUTION
CiC, p. 22, # 21, #22 (4 points)

CiC, p. 22, # 21.

a. The disease lasts 4 days, so the recovery coefficient b which is the reciprocal of the length of the illness is $b = 0.25$. The transmission coefficient a is the fraction of contacts per day between infecteds and susceptibles that are sufficient to spread the illness. In this case, $a = 0.003/6 = 0.0005$. Therefore the SIR model is

$$S' = -(0.0005)SI$$
$$I' = (0.0005)SI - 0.25I$$
$$R' = 0.25I$$

b. For the illness to fade away without becoming an epidemic, we need $I' = aSI - bI < 0$. Since $I > 0$, we need $aS - b < 0$. Thus $S < b/a = 500$. We require $S(0)$ to be below 500 individuals.

CiC, p. 22, # 22

a. The known formula for the threshold value of a generic S-I-R model is $S = b/a$.
b. Suppose we have two illnesses with the same transmission coefficient a, but different illness lengths λ_1 and λ_2. Suppose tht the first illness has the longer length, i.e., $\lambda_1 > \lambda_2$. Then for the recvoery coefficients of the two illnesses, we have $b_1 = 1/\lambda_1 < 1/\lambda_2 = b_2$.

   In the first illness, the threshold value is $S_1 = b_1/a$; in the second illness, it is $S_2 = b_2/a$. Since $b_1 < b_2$, we have $S_1 < S_2$. So the longer illness has a lower threshold value. This makes sense in that if the illness lasts longer, there are more chances for it to spread and so to avoid the epidemic, the size of $S(0)$ would have to be lower.