

Math 118 { Homework #3 (6 points)

- a. Use Euler's Method with $\Delta t = 1$ to obtain an approximation for $y(1) \approx 0.41$.

t	y	y'	Δy	Δt
0	0.41	0	0	1
1	0.41	XXX	XXX	XXX

Use Euler's Method $\Delta t = .5$ to obtain an approximation for $y(1) \approx 0.4510125$.

t	y	y'	Δy	Δt
0	0.41	0	0	.5
.5	0.41	.08025	.040125	.5
1.0	0.4510125	XXX	XXX	XXX

Use Euler's Method with $\Delta t = .25$ to obtain an approximation for $y(1) \approx 0.48078125$.

t	y	y'	Δy	Δt
0	0.41	0	0	.25
.25	0.41	.0200625	.005015625	.25
.50	0.415015625	.08025	.020625	.25
.75	0.435640625	.1805625	.045140625	.25
1.0	0.480781250	XXX	XXX	XXX

- b. The exact solution can be found by solving the IVP// $y' = 0.321t^2$) $y = .107t^3 + C$ and when $t = 0$, $y = 0.41$ so $C = 0.41$ so the exact solution is $y(t) = 0.41 + 0.107t^3$. The exact value of $y(1) = 0.1071^3 + .41 = 0.517$

The approximate answers are all under-estimates which get more accurate as Δt gets smaller. They are underestimates because the initial slope at $t = 0$ is zero and the exact solution we know is concave up at that point for $t > 0$.