Math 118 – Homework 2
CiC, p. 20-21, # 19, #20 (4 points)

CiC, p. 20-21, # 19.

$$S' = -aSI = -(0.00001)SI$$
$$I' = aSI - bI = (0.00001)SI - (1/14)I$$
$$R' = bI = (1/14)I$$

a. To cut in half the chance that a susceptible will fall ill, we divide the transmission coefficient by 2: $(0.00001)/2 = 0.000005$.

b. The threshold value of S is determined by the ratio of recovery coefficient/transmission coefficient. Thus if the transmission coefficient is cut in half, the threshold value is doubled. The new threshold value is $2 \times 7143 = 14,286$.

c. The quarantine does *not* eliminate the epidemic since $S(0) = 45,400$ is above the new threshold value of 14,286.

d. We need to find an a so that $S(0) = 45,400$ is below the value of b/a (where $b = 1/14$). So any value of a such that $a < 1.57 \times 10^{-6}$ will suffice.

e. If I never increases, then $I' \le 0$. and so $S \le b/a$. If $S(0) = 45,400$ and $b = 1/14$, then $a \le (1/14)/(45,400) = 1/635,600 \approx 1.57 \times 10^{-6}$. This is the largest value that a can have.
The original transmission coefficient was 0.00001. To reduce this to 1/635,600, we are taking the transmission coefficient down to about one-sixth of its original value.

CiC, p. 21, # 20.

a. In this model, $b = 0.08$, where b is the fraction of infecteds who recover per day. Then on average, the length of the illness is $1/b = 12.5$ days. This makes sense if we look at an "average" group of infected people. Some in this group will have just gotten ill (i.e., been sick one day), some will have been sick two days, and so on up to the subgroup who have been sick for over a week and are at 12.5 days of illness. If the infected people are spread evenly throughout the 12.5 days of illness, then on average, $1/12.5 = 0.08$ of them will recover per day. (See p.5 in CiC.)

b. For the number of infecteds to increase, we need $I' > 0$. This occurs when $aSI - bI = I(aS - b) > 0$ We assume that $I > 0$ and so we need to have $(aS - b) > 0$. This is the case when $S > b/a$.

c. Roughly, we have $b(100) = 0.08(100) = 8$ people recovering per day. So in one day (24 hours), 8 will recover.

d. If 30 new cases appear in a day, then S has decreased by 30 individuals that day. Thus the rate of change in S per day is $S' = -30$ for that day.

e. From (c.) and (d.), we can see that $-30 = S' = -aSI = -(0.00002)S(100)$. Solving for S, we find $S = 15,000$.