Math 118
Fall Term 2002

Final Exam Review

Derivative

Definition, algebraically and graphically. Necessity of local linearity.

Techniques of differentiation (product rule, chain rule, etc.)

Use of approximations to the derivative: $y' \approx \frac{\Delta y}{\Delta t}$

- Euler's method to approximate the solution of an IVP.
- Prediction of whether Euler approximation is underestimate/overestimte
- Estimation and error using the "Microscope Equation."
- 1^{st} order Taylor polynomial of $f(x)$ near $x = a$.
- Newton's method to find the root of an equation.

Information about the graph of a function from its derivatives

Optimization; finding max and min

Initial Value Problems (IVP)

Existence and Uniqueness Theorem

Checking a solution to an IVP

Information from a differential equation:

- Steady state, e.g., long term population in Logistic Growth Population model.
- Threshold value, e.g., SIR model.
- Sketching a solution and obtaining inflection points from $y' = f(x, y), y(a) = b$

Explicit solutions

- $y' = ky, \;\; y(0) = c$
  - population model
- $y' = c(y - k), \;\; y(0) = A$
  - Newton's Law of Cooling
- $y'' = -b^2 y, \;\; y(0) = A, \; y'(0) = B$
  - linear spring model
  - nonlinear spring model

Functions of two variables

Partial derivatives

Contour plots

Equation of a tangent plane; local planarity

Microscope equation and error estimation

Optimization; finding max and min

Constrained Optimization; **always check the boundary critical points**

Integrals

Techniques of integration (u–substitution, by parts, etc.)

Fundamental Theorem of Calculus, especially as it relates to IVPs

Applications of integration

- Accumulation Functions
- Cumulative probability distributions
- Arclength
- Area
- Volume

Summation techniques, and over/underestimation of area

- Right and left endpoint sum
- Midpoint sum
- Trapezoid sum
- Simpson's sum
- Using sums to estimate "un–antidifferentiable" definite integrals
- Estimating Midpoint, Trapezoid, Simpson's, Riemann Error
- Error control

Periodic functions

Amplitude, period, phase shift, combining sine and cosine functions

Modeling springs and pendulums

Conservation of energy and first integrals

Exact solution to linear spring motion IVP:

$y'' = -b^2 y, \;\; y(0) = a, \; y'(0) = p$

Series

Taylor polynomials and Taylor's Theorem

Taylor series

- Intervals of convergence
- Forming new series by substitution, differentiation, integration
- Using series to estimate "un–antidifferentiable" definite integrals
- Using Taylor approximations to determine the value of a limit
- Solving IVP using Taylor series/power series

Convergent and divergent series

- Geometric series
- Harmonic series
- P-series
- Alternating harmonic series
- Tests for convergence
  - Zero limit divergence test
  - Alternating series test
  - Ratio test
  - Integral test
  - Root test
  - Comparison test

Absolute Convergence Theorem: IF $\displaystyle\sum_{k=1}^{\infty} |a_k|$ **converges, THEN** $\displaystyle\sum_{k=1}^{\infty} a_k$ **converges**

Fourier polynomials and series