Derivative

Definition, algebraically and graphically. Necessity of local linearity.

Techniques of differentiation (product rule, chain rule, etc.)

Use of approximations to the derivative: $y' \approx \frac{\Delta y}{\Delta t}$

Euler’s method to approximate the solution of an IVP.

Prediction of whether Euler approximation is underestimate/overestimate

Estimation and error using the “Microscope Equation.”

1st order Taylor polynomial of $f(x)$ near $x = a$.

Newton’s method to find the root of an equation.

Information about the graph of a function from its derivatives

Optimization; finding max and min

Initial Value Problems (IVP)

Existence and Uniqueness Theorem

Checking a solution to an IVP

Information from a differential equation:

Steady state, e.g., long term population in Logistic Growth Population model.

Threshold value, e.g., SIR model.

Sketching a solution and obtaining inflection points from $y' = f(x, y), y(a) = b$

Explicit solutions

$y' = ky, \quad y(0) = c$

population model

$y' = c(y - k), \quad y(0) = A$

Newton’s Law of Cooling

$y'' = -b^2y, \quad y(0) = A, \quad y'(0) = B$

linear spring model

nonlinear spring model

Functions of two variables

Partial derivatives

Contour plots

Equation of a tangent plane; local planarity

Microscope equation and error estimation

Optimization; finding max and min

Constrained Optimization; always check the boundary critical points

Integrals

Techniques of integration (u-substitution, by parts, etc.)

Fundamental Theorem of Calculus, especially as it relates to IVPs
Applications of integration
  Accumulation Functions
  Cumulative probability distributions
  Arclength
  Area
  Volume

Summation techniques, and over/underestimation of area
  Right and left endpoint sum
  Midpoint sum
  Trapezoid sum
  Simpson’s sum
  Using sums to estimate “un–antidifferentiable” definite integrals
  Estimating Midpoint, Trapezoid, Simpson’s, Riemann Error
  Error control

Periodic functions
  Amplitude, period, phase shift, combining sine and cosine functions
  Modeling springs and pendulums
  Conservation of energy and first integrals
  Exact solution to linear spring motion IVP:
  \[ y'' = -b^2 y, \quad y(0) = a, \quad y'(0) = p \]

Series
  Taylor polynomials and Taylor’s Theorem
  Taylor series
    Intervals of convergence
    Forming new series by substitution, differentiation, integration
    Using series to estimate “un–antidifferentiable” definite integrals
    Using Taylor approximations to determine the value of a limit
    Solving IVP using Taylor series/power series

Convergent and divergent series
  Geometric series
  Harmonic series
  P-series
  Alternating harmonic series
  Tests for convergence
    Zero limit divergence test
    Alternating series test
    Ratio test
    Integral test
    Root test
    Comparison test

**Absolute Convergence Theorem:** IF \( \sum_{k=1}^{\infty} |a_k| \) converges, THEN \( \sum_{k=1}^{\infty} a_k \) converges

Fourier polynomials and series