We want to know $\frac{d\theta}{dt}$ when $x = 0$ and $\frac{dx}{dt} = -130 \text{ ft/s}$.

An equation relating $x$ and $\theta$ is:

$$\tan \theta = \frac{x}{2}$$

Differentiating both sides:

$$\frac{d}{dt} \tan \theta = \frac{d}{dt} \frac{x}{2}$$

$$\sec^2 \theta \cdot \frac{d\theta}{dt} = \frac{1}{2} \cdot \frac{dx}{dt}$$

Using $\frac{1}{\sec^2 \theta} = \cos^2 \theta$

$$\frac{d\theta}{dt} = \frac{1}{2} \cdot \frac{dx}{dt} \cdot \cos^2 \theta$$

*When $x = 0$, $\theta = 0 \Rightarrow \cos^2 \theta = 1$.* So,

using the relevant values:

$$\frac{d\theta}{dt} = \frac{1}{2} (-130) \cdot 1 = -65 \text{ rad/s}$$

The player's eyes must move at a rate of $-65 \text{ rad/s}$.
30. We are in the same situation as problem 29, but now we're given \( \frac{d\theta}{dt} = \frac{\pi}{2} \) and we want the corresponding \( \frac{dx}{dt} \).

So: \( \frac{d\theta}{dt} = \frac{1}{2} \frac{dx}{dt} \) (from 29)

\[
\frac{\pi}{2} = \frac{1}{2} \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \pi \text{ ft/s}
\]

The fastest pitch you could watch cross home plate (while maintaining focus!) moves at \( \pi \text{ ft/s} \).

51. We know: \( \frac{dy}{dt} = 2 \text{ ft/s} \)

\( h = 6 \text{ ft} \) (and is constant)

We want to know \( \frac{dx}{dt} \) when \( x = 20 \), \( x = 10 \).

A relationship between \( x \) and \( y \) and \( h \) is:

\[
x^2 + h^2 = y^2 \text{ (Pythagorean's theorem)}
\]

\[
2x \frac{dx}{dt} + 0 = 2y \frac{dy}{dt} \text{ since } h \text{ is constant.}
\]

When \( x = 20 \) we need \( y \):

\[
20^2 + 6^2 = y^2
\]

\[
436 = y^2 \Rightarrow y = 20.88 \text{ ft}
\]

When \( x = 10 \) we need \( y \):

\[
10^2 + 6^2 = y^2
\]

\[
136 = y^2 \Rightarrow y = 11.66 \text{ ft}
\]

When \( x = 20 \):

\[
2(20) \frac{dx}{dt} = 2(20.88) \cdot 2
\]

\[
\frac{dx}{dt} = 2.088 \text{ ft/s}
\]

At 20 ft away from the dock, the boat's speed is \( \approx 2.088 \text{ ft/s} \).
Because the inverse function is the reflection of $f(x)$ across the line $x=y$, it will be concave up.

Because the inverse function is the reflection of $f(x)$ across the line $x=y$, it will be concave down.

6.7: 5, 6

5. $\sin^{-1}(0)$

$\sin(0) = 0$  
$\Rightarrow \sin^{-1}(0) = 0$

6. $\cos^{-1}(0)$

$\cos(\pi/2) = 0$  
$\Rightarrow \cos^{-1}(0) = \pi/2$
When $x = 10$:

$$2(10) \frac{dx}{dt} = 2(11.66) \cdot 2$$
$$\frac{dx}{dt} = 2.332 \text{ ft/s}$$

At 10 ft away from the dock, the boat's speed is 2.332 ft/s.

**SHEET SOLUTIONS AT END**

SM Sec 6.2: 20, 24, 35, 36

20. $f(x) = x^5 + 4$:

The graph of $f(x)$ looks like:

The function is 1-1
(it passes the horizontal line test) so it has an inverse.

To solve for the inverse:

$$x = y^5 + 4$$

$$x - 4 = y^5 \Rightarrow f^{-1}(x) = \sqrt[5]{x - 4}$$

24. $f(x) = \sqrt{x^2 + 1}$

The graph of $f(x)$ looks like:

The function is not 1-1
so it does not have an inverse on the domain of real numbers.
(if you restricted its domain it would)
1. From Calculus for the Life Sciences by Greenwell, Ritchey and Lial; Example 5. Blood flows faster the closer it is to the center of a blood vessel because of the reduced friction with cell walls. According to Poiseuille's laws, the velocity $V$ of blood is given by

$$V = k(R^2 - r^2),$$

where $R$ is the radius of the blood vessel, $r$ is the distance of a layer of blood flow from the center of the vessel, and $k$ is a constant, assumed here to equal 375. Suppose a skier's blood vessel has radius $R = 0.08$ millimeter and that cold weather is causing the vessel to contract at a rate of $dR/dt = -0.01$ millimeter per minute. How fast is the velocity of the blood changing?

**hint:** treat $r$ as constant!

We are interested in $\frac{dV}{dt}$ when $R = 0.08$, $\frac{dR}{dt} = -0.01$ mm/min.

Use the equation above and differentiate both sides:

$$\frac{dV}{dt} = k \left( 2R \frac{dR}{dt} - 0 \right)$$

$$= 2kR \frac{dR}{dt}$$

So, with our relevant values

$$\frac{dV}{dt} = 2(375)(0.08)(-0.01)$$

$$\frac{dV}{dt} = -0.6 \text{ mm/min}$$

The velocity of the blood is decreasing at a rate of $-0.6$ mm/min.
2. From Calculus for the Life Sciences by Greenwell, Ritchey and Lial; Problem 17. Sociologists have found that crime rates are influenced by temperature. In a midwestern town of 100,000 people, the crime rate has been approximated as

\[ C = \frac{1}{10} (T - 60)^2 + 100, \]

where \( C \) is the number of crimes per month and \( T \) is the average monthly temperature in degrees Fahrenheit. The average temperature for May was 76°, and by the end of May the temperature was rising at the rate of 8° per month. How fast is the crime rate rising at the end of May?

We are interested in \( \frac{dc}{dt} \) when \( T = 76^\circ \) and \( \frac{dT}{dt} = 8^\circ/\text{mo} \):

Using our equation above and differentiating both sides:

\[ \frac{dc}{dt} = \frac{1}{10} \cdot 2 (T - 60) \frac{dT}{dt} + 0 \]

\[ = \frac{1}{5} (T - 60) \frac{dT}{dt} \]

Using our relevant values:

\[ \frac{dc}{dt} = \frac{1}{5} (76 - 60)(8^\circ) = 25.6 \text{ crimes/month} \]

The crime rate is increasing by 25.6 crimes/month.