Math 114: Calculus I **Final Exam** Name: ____________________

Instructor: Ramin Naimi Monday, 13 Dec 2004

Closed book. Closed notes. NO CALCULATORS. Please write very legibly. You may use the back of each sheet for extra space. Circle all scratch work and write "Do not grade" on it.

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1. (a) Newton's law of cooling (and warming) states that the rate of change of an object's temperature is proportional to the difference between the object's temperature and the ambient temperature. Using T for the object's temperature, and R for ambient temperature (assumed to be constant), translate this into a differential equation (rate equation), and then solve the differential equation. (You should get $T = Ae^{kt} + R$.) Show and explain all work.

   (b) If a cup of coffee's temperature is 100° ten minutes after being poured, its initial temperature (immediately after being poured) was 150°, and room temperature is 70° and constant, find the coffee's temperature twenty minutes after being poured. Show and explain all work.

2. (a) Find the derivative of $f(x) = e^{|x|}$. Show and explain all work.

   (b) For which x is $f(x)$ differentiable? Why?

3. (a) Give the definition of a *separable* differential equation.

   (b) Determine which of the following differential equations is separable. Do not find a solution for any of them. Show and explain all work.

      i. $y' + x^2 + 2xy = (x + y)^2$

      ii. $y' + x^2 = (x + y)^2$

      iii. $y' + x = x^2 + y$

4. (a) Use Euler's Method with $\Delta x = 2$ to estimate $y(5)$, given $y' = y + x$ and $y(1) = 0$. Show all work and all computations clearly.

   (b) Is your answer an over- or under-estimate or neither for the actual value of $y(5)$? Why? (Hint: Is y' constant, increasing, or decreasing?)

5. Suppose $f(x)$ is a differentiable function such that $f(5) = 7$, $f'(5) = 2$, and $f''(5) = 4$.

   (a) Find one of $(f^{-1})'(5)$ or $(f^{-1})'(7)$ or $(f^{-1})'(2)$. State which one you are finding.

   (b) Use a linear approximation to estimate $f(5.3)$.

   (c) Use a linear approximation to estimate $f'(4.8)$.

6. Find each limit. Show all work, and explain when appropriate.

   (a) $\displaystyle\lim_{x\to\infty} \frac{\sin x + 0.5}{x}$

   (b) $\displaystyle\lim_{x\to 0^+} \frac{x + \ln x}{e^x}$

   (c) $\displaystyle\lim_{x\to 0} \sqrt{x}$

   (d) $\displaystyle\lim_{x\to -\infty} (\frac{1}{x} + x)$

   (e) $\displaystyle\lim_{x\to 1} \frac{x - 1}{\cos(x - 1) - 1}$

(f) $\lim_{x\to\infty} (\ln x)\sin(1/x)$

7. Let $f(x) = \dfrac{x^2}{x-1}$. Show (and explain, when appropriate) all work for the following.

   (a) Find $f'(x)$ and $f''(x)$. Simplify your answers as much as you can. (You may check your answers with me before continuing to the next part.)

   (b) Find all critical points and local extrema of f.

   (c) Find where f is concave up or down, and find all inflection points.

   (d) Find the domain of f; its intercepts; and all horizontal and vertical asymptotes by taking appropriate limits.

   (e) Graph $f(x)$ (use the back of this sheet, please). Your graph should be consistent with all your answers above.

8. Do **only one** of the following two problems:

   (a) We want to cut a rope of length L into two pieces and make a circle with each piece, with the goal of enclosing the largest combined total area with the circles. Where should we cut the rope, or is it better to not cut at all and create only one circle? Show and explain all work.

   (b) You are standing in front of a picture hanging on a wall. The bottom of the picture is two feet above your eye-level. The picture is three feet tall. At what distance should you stand from the wall so as to maximize the subtended viewing angle (from the top to the bottom of the picture)? Show and explain all work. Hint: Recall that $[\arctan x]' = 1/(1+x^2)$.