Class 24 HW SOLUTIONS **to selected problems**. Math 110, Fall 98
Was due Friday, 11/06/98: HH, section 4.6: 33, 34; section 4.7: 21.

33. (a) $z^2 = 0.5^2 + x^2$, so $z = \sqrt{0.5^2 + x^2}$.

(b) You can differentiate either of the above two equations. Let's use the second one here:
$dz/dt = (1/2)(0.25 + x^2)^{-1/2} 2x(dx/dt)$; in order to plug in values, we need to find what x is when $z = 1$ kilometer.
$1^2 = 0.5^2 + x^2$, so $x = \sqrt{0.75}$. So we get $dz/dt = (1/2)(0.25+0.75)^{-1/2} 2\sqrt{0.75}(0.8) = 0.4\sqrt{0.75}$.
(c) Let θ be the angle between the line from the camera to the train, and the line from the camera perpendicular to the railroad tracks.
Then, we're looking for $d\theta/dt$.
From the picture in the book we see that $\tan(\theta) = x/.5 = 2x$.
So, $\sec^2(\theta)\ d\theta/dt = 2\ dx/dt = 2(0.8) = 1.6$.
Now, $\cos(\theta) = 0.5/z = 0.5/1 = 0.5$, so $\sec^2(\theta) = (1/0.5)^2 = 4$.
So, $d\theta/dt = 1.6/4 = 0.4$ radians per minute.

34. $V = (4/3)\pi r^3$, so $dv/dt = 4\pi r^2\ dr/dt = 4\pi 10^2(2) = 800\pi$.

21. $y = x^2$, so $y' = 2x$. So at $x = 1$ we get $y' = 2$.
So, the equation of the line tangent to the parabola at the point $(1, 1^2)$ is: $y - 1 = 2(x - 1)$.
Let's call this line L. Let P denote the point where the circle is tangent to L. The line going through the center of the circle and the point P is perpendicular to L, so its slope is the negative reciprocal of the slope of L. So its slope is -0.5. We also know that this line goes through the point (8,0), the center. So we find the equation of this line, then find where it intersects L, and from that use the distance formula to find the radius. (For more details see class notes, as this problem was solved in detail in the class.)