Preparing for THE FINAL EXAM

1. **THE IDEAS ARE THE MOST IMPORTANT THING!** And what are those ideas? A partial list is:

- **Initial Value Problems**—You should know what an IVP is and construct a simple IVP given specific information. For example, if we say that a value P changes at a rate proportional to the square root of P, and that $P(2) = 3$, the IVP would be

$$\begin{aligned} P' &= k\sqrt{P} \\ P(2) &= 3. \end{aligned}$$

  The most complex IVP we expect you to be comfortable with is the SIR model of disease. You should be able to examine rate equations to gain intrinsic information about an IVP, e.g. the threshold value for an SIR model.

- **Solutions to IVP's**—You should know what a solution for an IVP is (a function or functions), but you should also know that we may not be able to express the solution explicitly.
- **Euler's method** is used to *approximate* solutions for IVP's. You should be comfortable using Euler's method to approximate the solution for a given IVP. For example, with the IVP above if we set Δt, you should be able to set up the table and compute estimates for $P(t)$ for $2, 2+\Delta t, 2+2\Delta t, 2+3\Delta t, \ldots$. You should be able to find an estimate for $P(t)$ for any t in the domain of our approximating function.
- **The SIR model** should be very familiar. You should feel comfortable adjusting a given SIR model to reflect new situations.
- **Functions.** You should feel comfortable taking the graph of a basic function and transforming it, including translating, stretching and compressing. You should be able to determine whether any given function is *even* or *odd* or *neither* or *both*. You should be able to recall and discuss the six ways of viewing a function.
- **Lines.** You should be able to come up with the equation of a line, given sufficient information. Equations can be in any appropriate form. Once you have the equation for the line, you should be able to find the corresponding dependent value,given the independent value.
- **Successive Approximations.** What does it mean to say that successive approximations converge and where have we used this technique?
- The two sorts of *Local Approximations*, namely

$$\begin{aligned} \Delta f &\approx f'(a)\Delta x \qquad \text{Microscope Equation} \qquad (1) \\ \text{and } f'(a) &\approx \frac{\Delta f}{\Delta x} \qquad \text{Difference Quotient.} \qquad (2) \end{aligned}$$

  You should know what each approximation is used for and what must be known in order to use each approximation.

- **Tangent Line** to a curve at a point. You should be able to use the tangent line to estimate values of the function near the point. You should also be able to write the equation for the tangent line to a curve at a point.
- **Limit Definition of the Instantaneous Rate of Change** of a function at a point. We extend this local definition for all values in the domain of the function, yeilding the
- **Limit Definition for the Derivative Function** for a function. This leads us to examine more global aspects of functions. In particular, the
- **Relationship between the Graphs** of the derivative function and the original function.
- **Critical points, inflection points, global and local extrema** Given a function, you should be able to obtain crucial information about it which allows you to sketch it.
- **Optimization.** You should be able to set up and solve simple optimization problems. This typically involves using the First and Second Derivative Tests to identify critical points and local extrema.
- **Continuity and Differentiability.** You should know the meanings of the terms "continuous" and "differentiable" and be able to determine and identify whether a function is either, neither or both, at a particular point.
- Finally, you should know the derivatives of the *Elementary Functions* and the *Rules of Differentiation*. These include the *Power Rule*, the *Product Rule*, the *Quotient Rule* and the *Chain Rule*.

- **Implicit Differentiation** Besides taking the derivative $\frac{dy}{dx}$ of a function $y = f(x)$ explicitly you should be able to differentiate functions which can NOT be written as explicit functions of x, but have the form $f(x, y) = c$ instead.
- **Related Rates** You should be able to apply The Chain Rule to compute how a function of multiple variables has derivatives related to each other. In other words, you have a function $F(x, y)$ and you have $x(t)$ and $y(t)$ then $\dfrac{dF}{dt} = \dfrac{dF}{dx}\dfrac{dx}{dt} + \dfrac{dF}{dy}\dfrac{dy}{dt}$
- **Newton's Method** A useful iterative technique for solving equations of the form $f(x) =$. You should know theiterative step $x_{n+1} = x_n - f(x_n)/f'(x_n)$ and how to use it to find a sequence of approximations which produce a solution to the equation.
- **Limits** You should understand what the possible values of a limit can be (a number, $+\infty$, $-\infty$ or DNE). You should also understand limits from the left and limits from the right. You shold understand that a limit is a link between two *processes* or *trends*. It says that as one sequence of numbers gets closer and closer to a particular value, this other related sequence of numbers also hopefully gets closer and closer to some other number, which we call the limit.
- **L'Hôpital's Rule** For indeterminate limits of the form $\frac{\infty}{\infty}$, $\frac{0}{0}$ and $0 \cdot \infty$ you should know how to apply L'Hôpital's rule.
- **Continuity and Differentiability** You should know when the rules are that you can evaluate a limit $\lim\limits_{x \to c} f(x)$ by just using direct substitution, i.e. $f(c)$. A function is said to be continuous if you can evaluate a limit in such a manner. A function is not differentiable at a point if the derivative does not exist at that point, which would mean that the limit of difference quotients does not exist at that point.
- **Horizontal and Vertical Asymptotes** You should know what limits you need to take in order to determine whether a function $f(x)$ has horizontal asymptotes. Also, you should know how you find vertical asymptotes.
- **Curve Sketching** You should be able to apply your knowledge of derivatives to find intervals on which the function is positive/negative, increasing/decreasing, concave up/concave down, as well as determining where the asymptotes are.
- **Inverse Functions**
  You should be able to determine whether a function has an inverse (Horizontal Line Test) and also be able to compute the inverse function and denote it using the correct variable.You should be able to have a graphical sense of what the inverse of a function looks like. You should also be able to find the derivative of an inverse at a point inrelation to the derivative of the original function.

2. **Problems will closely resemble the practice exam questions.** But they will not be identical to these. An excellent way to study is to review the three tests and to do the practice exam (it is basically the 1997 Final Exam.)
3. **Don't forget about labs.** The story being told in lab closely parallels the story being told in class.

Rules for the Exam

1. **Blue Notes:** You are allowed the ONE sheet of paper for written notes. Only the use of notes on **one side** of this sheet of paper are permitted during the exam. You may not use the program function of your calculator to store additional notes. There will be fewer problems involving simple calculations and more involving interpretations of the main ideas, i.e. short answer questions.
2. There will be a pledge on the exam. By signing the pledge, you indicate that you followed all the rules of this exam.
3. No answer will be given credit without accompanying work. **No exceptions.** Unless otherwise indicated, answers should be left in exact form, i.e. no decimal approximations.
4. This list of rules is not necessarily exhaustive. If you have any questions about what is allowed and what is not, you are responsible for asking me. Ignorance is not an excuse.
5. The final exam is on Thursday, from 6:30-9:30pm in Fowler Hall. Both sections take the exam at the same time. You will have the full three hours to complete the exam, though we will design it to be completed in two hours. Good luck!