1. Use the rules of differentiation to find the first derivative for the following functions. **DO NOT SIMPLIFY YOUR ANSWER.**

   (a) $f(t) = t^{7/2} - 3t + 5$

   (b) $y = \dfrac{e^x}{x}$

   (c) $g(x) = \sqrt[3]{(1-x^3)^2}$

   (d) $z = 2^x \log_3 x$

   (e) $h(x) = \dfrac{\ln(\sin(2x))}{\tan(x)}$

2. Use the **limit definition** of the derivative to find the derivative of the function $f(x) = \dfrac{1}{x-2}$. You must **show all of your work** in order to get **any credit** for this problem.

3. Find the equation for the tangent line to the curve $q(x) = \dfrac{3^x}{x}$ at the point $(1, 3)$.

4. What is the microscope equation (i.e. equation of tangent line) for $y = \sin(x)$ at the point $\left(\dfrac{\pi}{12}, \dfrac{\sqrt{6}-\sqrt{2}}{4}\right)$?

   Use this microscope equation to estimate $\sin\left(\dfrac{1}{4}\right)$.

5. A *Calculus in Context* textbook has been dropped from a cliff. Using the table of values given below, make an estimate of the instantaneous velocity of the textbook at time $t = 2$ seconds. How many decimal places of accuracy does your estimate exhibit? Justify your answer in writing.

Time t	Height h(t)
2.0000	936.00000000
2.0001	935.99359984
2.0010	935.93598400
2.0100	935.35840000
2.1000	929.44000000
3.0000	856.00000000

6. Consider the equation $y^2 + xy - x^2 = 5$.

   (a) Explain why there is no derivative function $f'(x)$ for the equation above.

   (b) Find $\dfrac{dy}{dx}$ at the point $(1, 2)$ and also at the point $(1, -3)$.

7. Show that the function $g(x) = 2x + \sin(x)$ is one-to-one and compute $[g^{-1}]'(4\pi)$.

8. Find the inverse function $f^{-1}(x)$ for the function $f(x) = x^2 - 3$ on the interval $[0, +\infty)$ and indicate the domain of $f^{-1}(x)$.

9. Find the following limits (if they exist):
   (a) $\lim\limits_{x\to 0+} x^{x^x}$ **(b)** $\lim\limits_{x\to 1} \dfrac{1-x^3}{1-x}$ **(c)** $\lim\limits_{x\to\infty} 5^x - 2^x$ **(d)** $\lim\limits_{x\to 0-} \dfrac{.00001}{x^2} - \dfrac{10000}{x}$ **(e)** $\lim\limits_{x\to 0+} \dfrac{.00001}{x^2} - \dfrac{10000}{x}$

10. Below is a simple polynomial function and its first two derivatives (in factored form)

$$\begin{aligned} f(x) &= x^4 - 4x^3 + 16x \\ f'(x) &= 4(x+1)(x-2)^2 \\ f''(x) &= 12x(x-2) \end{aligned}$$

   (a) Determine on what intervals the function $f(x)$ is increasing.

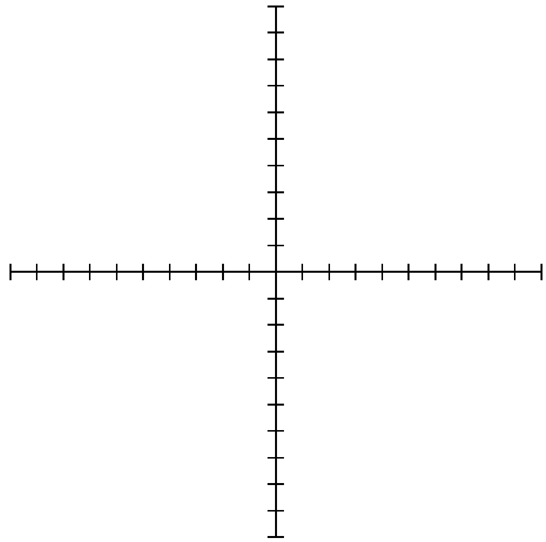
   (b) Determine where the function $f(x)$ has relative extrema.

   (c) Determine on what intervals the function $f(x)$ is concave down.

   (d) Determine where the function $f(x)$ has points of inflection.

   (e) One of the x-intercepts of the function $f(x)$ is $x = 0$. The other one lies between -2 and -1. Use Newton's Method with an initial guess of $x_1 = -2$ to find the other intercept to two decimal places.

   (f) Graph the function $f(x)$ below, labeling your scale and all important points (intercepts, extrema, points of inflection, etc.).



11. A woman 1.8 meters tall walks at a rate of 1.2 meters per second away from a lamp which is 3 meters above the ground (see the figure). When she is 4 meters from the base of the lamp, at what rate is the tip of her shadow moving along the ground?

12. Consider the Initial Value Problem (IVP)

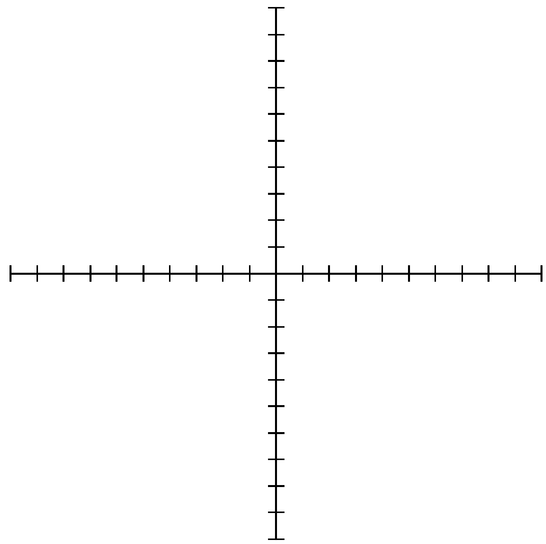
$$\begin{aligned} C' &= -2C \\ C(0) &= 1. \end{aligned}$$

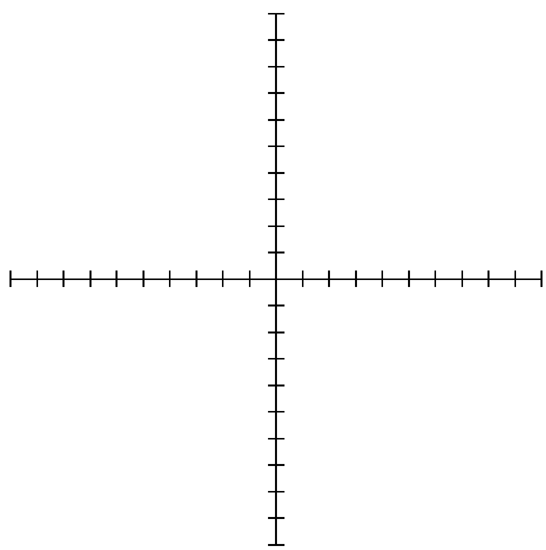
(a) Use Euler's method with a time step of $\Delta t = \dfrac{1}{10}$ to fill in the empty boxes in the table below and **find an estimate of** $C\left(\frac{1}{3}\right)$**.**

t	C	C'	ΔC
0	1	XXXXXXXX XXXXXXXX XXXXXXXX	XXXXXXXX XXXXXXXX XXXXXXXX
0.1	$\frac{4}{5}$		
0.2			
0.3			
0.4	$\frac{256}{625}$	XXXXXXXX XXXXXXXX XXXXXXXX	XXXXXXXX XXXXXXXX XXXXXXXX

(b) Verify that $c(x) = e^{-2t}$ is the solution to the IVP above. Show all of your work. Then use your calculator to compute $c\left(\frac{1}{3}\right)$.

13. Sketch a possible derivative function for the function below.





14. Determine if the function $h(x) = \dfrac{3x^2+1}{2x^2-x}$ has any vertical or horizontal asymptotes by evaluating the appropriate limits. If it does have vertical or horizontal asymptotes, give the equations for those asymptotes.

15. On the interval $[-1, 2]$, find the x-values where the global (or absolute) extrema occur for the function $f(x) = x^4 - 4x^3 + 3x^2$.