Practice exam #3. Math 110. Fall 1998.

1. (a) Implicitly differentiate each of the following:

$$\frac{d}{dx}\left[x^2 + y^2 + \sin(xy)\right] =$$

$$\frac{d}{dy}\left[x^2 + y^2 + \sin(xy)\right] =$$

(b) Find the slope of the curve $y + x\ln(y) = (x+1)\ln(x+1) + 1$ at $x = 0$.

(c) Find the second derivative d^2y/dx^2 for the implicit function $x^2 + y^2 = 1$.

2. A ladder is leaning against a wall. The bottom of the ladder is slipping away from the wall at a constant rate of 2 ft/sec. Does a person standing on top of the ladder fall faster and faster, or slower and slower, or at a constant rate, as the ladder slips? Justify your answer by using the methods of implicit differentiation and related rates of change.

3. (a) Sketch a graph of the function $f(x) = (x-1)^2 - 3$.

(b) Suppose we're trying to find the two roots of this function using Newton's Method. If we start with $x = 2$ as our initial guess for Newton's Method, which of the two roots will we get? Explain why this happens.

(c) If we start with $x = 1$ as our initial guess for Newton's Method, which of the two roots will we get? Explain PICTORIALLY why this happens.

4. Evaluate the following limits. You may use L'Hopital's rule where appropriate. Show all work.

(a) $\lim_{x\to 1} x^2/e^x$. (b) $\lim_{x\to\infty} x^2/e^x$.

(c) $\lim_{x\to-\infty} x^2/e^x$. (d) $\lim_{x\to 0^+} \ln(x)/e^x$.

(e) $\lim_{x\to 1} \ln(x)/(x-1)$. (f) $\lim_{x\to 0^+} \ln(x)/\sin(x)$.

(g) $\lim_{x\to 2} 1/\ln(2-x)$. (h) $\lim_{x\to 0} \ln(x)/\sin(x)$.

(i) $\lim_{x\to 0} x/x$. (j) $\lim_{x\to 0} x/|x|$.

(k) $\lim_{x\to 0} x^{2x}$. (l) $\lim_{x\to 0} \cos(x^{2x})$.

5. (a) Find all horizontal and vertical asymptotes of the function $f(x) = \dfrac{e^x}{x^{1000}}$.

(b) Find the domain and all critical points of $f(x)$.

(c) Find all local and global extrema of $f(x)$.

(d) Use the above information to sketch a graph of $f(x)$. Be careful, your graphing calculator will very easily mislead you in this problem!