Basic Calculus I

Class 19

Friday October 19

Chain Rule and Related Rates

Exercise

Anton, Bivens & Davis **3.6.71**. Consider the following table of values

x	$f(x)$	$f'(x)$
2	1	7
8	5	-3

(a) Find $g'(2)$ where $g(x) = [f(x)]^3$

(b) Find $h'(2)$ where $h(x) = f(x^3)$

Most examples of the Chain Rule actually appear in real-world application problems such as the one below.

EXAMPLE

Anton, Bivens & Davis **3.6.66**. The force F (in pounds) acting at an angle θ with the hotizontal that is needed to drag a crate weighing W pounds along a horizontal surface at a constant velocity is given by

$$F = \frac{\mu W}{\cos(\theta) + \sin(\theta)}$$

where μ is called the *coefficient of sliding friction* between the crate and the surface. Suppose the crate weights 150 lb and that $\mu = 0.3$.

(a) Find $\dfrac{dF}{d\theta}$ when $\theta = 30^{\mathrm{O}}$. Express the answer in units of pounds/degree.

(b) Find $\dfrac{dF}{dt}$ when $\theta = 30^0$ if θ is decreasing at the rate of 0.5^{O}/s at this instant.

Related Rates

The most common application of the chain rule is when two variables x and y which are related through a function $y = f(x)$ each depend on time, so that $x = x(t)$ and $y = y(t)$. The rate at which y changes with respect to t, $\frac{dy}{dt}$ is related to the rate at which x changes with respect to t, $\frac{dx}{dt}$, by the Chain Rule:

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

EXAMPLE

A 10-foot ladder stands on a horizontal floor and leans against a vertical wall. Use x to denote the distance along the floor from the wall to the ladder, and use y to denote the distance along the wall from the floor to the top of the ladder. If the foot of the ladder is dragged away from the wall, find an equation that relates rates of change of x and y with respect to time.

Exercise

Anton, Bivens & Davis **3.7.14**. A spherical balloon is inflated so that its volume is increasing at the rate of 3 ft^3/min. How fast is the diameter increasing when the radius is 1 ft?

Algorithm for Solving Related Rates Problems

- Step 1 Assign letters to all the quantities that vary with time and any others that seem relevant. Give a definition for each letter.
- Step 2 Identify the rates of change that are known and the rate of change that is unknown. Interpret each rate as a derivative.
- Step 3 Find an equation that relates the variables whose rates of change were identified in Step 2.
- Step 4 Differentiate both sides of the equation found in Step 3 with respect to time to produce a relationship between the known rates of change and the unknown rates of change.
- Step 5 AFTER completing Step 4, substitute all the known values for the rates of change and the variables, and then solve for the unknown rate of change.