# Basic Calculus I

*Class 18*

Wednesday October 17

**Trigonometric Functions and the Chain Rule**

---

EXAMPLE

Recall two standard limits that we learned way back in *Class 12*

$$\lim_{h\to 0}\frac{\sin(h)}{h}=1 \text{ and } \lim_{h\to 0}\frac{1-\cos(h)}{h}=0$$

Now that we know the limit definition of the derivative (and recalling $\cos(0)=1$ and $\sin(0)=0$) we can re-write these limits as

$$\lim_{h\to 0}\frac{\sin(h)-\sin(0)}{h}=1 \text{ and } \lim_{h\to 0}\frac{-\cos(h)-(-\cos(0))}{h}=0$$

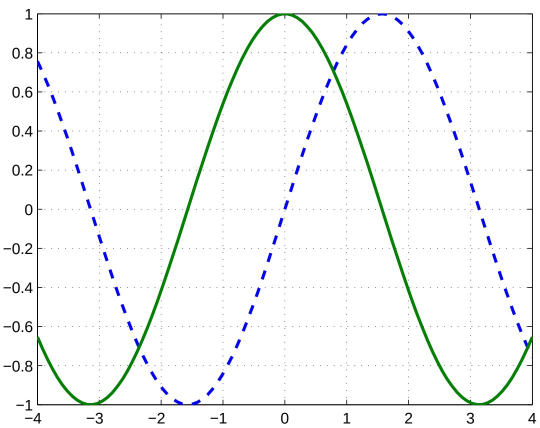
Compare these limits to: $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

Each of our initial two limits can be written as the derivative of what function $f(x)$ evaluated at what point $a$?

$\lim_{h\to 0}\frac{\sin(h)}{h}=$ ____________________ and $\lim_{h\to 0}\frac{1-\cos(h)}{h}=$ ____________________

**Exercise**

Consider the following graph of a function $f(x)$ and its derivative $f'(x)$. Can you identify which graph is $f(x)$ and which is $f'(x)$?




**THEOREM: Derivatives of Trigonometric Functions**

Here are some derivatives we need to know

**(a)** $\frac{d}{dx}[\sin(x)]=\cos(x)$ **(b)** $\frac{d}{dx}[\cos(x)]=-\sin(x)$ **(c)** $\frac{d}{dx}[\tan(x)]=\sec^2(x)$

**(d)** $\frac{d}{dx}[\cot(x)]=-\csc^2(x)$ **(e)** $\frac{d}{dx}[\sec(x)]=\sec(x)\tan(x)$ **(f)** $\frac{d}{dx}[\tan(x)]=-\csc(x)\cot(x)$

**EXAMPLE**

We can reproduce any one of the formulas (c)-(f) by using (a) and (b) and various Derivative Rules.

**THEOREM: Chain Rule**

IF $g$ is differentiable at $x$ and $f$ is differentiable at $g(x)$ THEN the composition of $f$ and $g$, $f \circ g$, is differentiable at $x$ and given by $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

The Chain Rule is often written as

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

where $u = g(x)$ so that $y = (f \circ g)(x) = f(g(x)) = f(u)$

The Chain Rule is the most important of the derivative rules that we have learned so far because it applies to so many different functions and situations.

**EXAMPLE**

Evaluate $\frac{d}{dx}[\sqrt{x^3 + 2x + 7}]$.

GROUPWORK

Consider $f_1(x) = \sin(x^2)$, $f_2(x) = \sin^2(x)$, and $f_3(x) = \sin(x)x^2$. Find the derivatives of each function and state what derivative rule(s) you are using in each case.

Notice how "similar" each of the functions $f_1$, $f_2$ and $f_3$ are but how different their derivatives are.