Math 110 — **BASIC CALCULUS I** — Fall 2007

Class 17

Wednesday October 10

Rules of Differentiation

The Product Rule

If $f'(x)$ and $g'(x)$ exist then the derivative of the product $f(x) \cdot g(x)$ exists and is given by the formula $\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)]g(x) + f(x)\frac{d}{dx}[g(x)]$

EXAMPLE

We can prove the Product Rule by using the limit definition of the derivative and some algebra.

The Quotient Rule

If $f'(x)$ and $g'(x)$ exist then the derivative of the quotient $\frac{f(x)}{g(x)}$ exists and is given by the formula $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$ (We will not prove the quotient rule, but the textbook does, on page 200-201.)

Exercise

Use the Quotient Rule to show the derivative $\frac{d}{dx}\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{[g(x)]^2}$

EXAMPLE

We'll use the above rule (sometimes called **the Reciprocal Rule**) combined with the Product Rule to generate the Quotient Rule.

GroupWork

1. Evaluate $\frac{d}{dx}[(4-x^3)(x^2+x-1)]$.

2. $f(x) = \left[\frac{3x+4}{x^2+1}\right]$, find $f'(x)$.

3. Evaluate $D_x^4\left[x^{-2}+x^9\right]$

4. *Anton, Bivens & Davis*, **Question 3.4.30.**
Find all values of a such that the curves $y = a/(x-1)$ and $y = x^2+2x+1$ intersect at right angles.