Class 14

Wednesday October 3

Introducing The Derivative

DEFINITION: Derivative

The slope of a function $f(x)$ at a point $x = a$ is also called **the derivative** of $f(x)$ at $x = a$. This is denoted by the symbols $f'(a)$.

EXAMPLE

Let $f(x) = x^2 - 1$.

1. $f(2) =$

2. Use the table below to estimate the derivative of $f(x) = x^2 - 1$ at $x = 2$.

x	$f(x)$	$x - 2$	$f(x) - f(2)$	estimate for derivative
1				
1.5				
1.9				
1.99				
1.999				

So, the DERIVATIVE of $f(x) = x^2 - 1$ at $x = 2$ is EXACTLY

The mathematical way of abbreviating this long sentence is:

3. There is also a third name for *slope* and *derivative*: ______________________________.

So we denote the derivative of f at $x = 2$ by the symbols ______.

Note:

$\frac{f(x) - f(2)}{x - 2}$ is called the ________ rate of change of f **over** the interval ________,

while

$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$ is called the ____________ rate of change of f **at** ________.

Exercise

Let's find the EXACT slope (derivative) of $f(x) = x^2 - 1$ at $x = 2$.

Step 1. Simplify the difference quotient.

$$\frac{f(x) - f(2)}{x - 2} =$$

Step 2. "Take the limit."
What happens to your answer in Step 1 as x gets closer and closer to 2 ?

We write: $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} x + 2 =$

DEFINITION: Tangent Line

The tangent line to the graph of f at any point $(a, f(a))$ exists if $f'(a)$ exists, and the slope of this *tangent line* has the value $f'(a)$.

Exercise

Find the slope of the line tangent to the graph of $f(x) = 3x^2$ at $x = 1$.
Step 1. Simplify.

Step 2. Take the limit.

GroupWork

4. Consider a graph of the parabola $f(x) = 3x^2$.
(a) Find the equation of the line tangent to the graph of the parabola at $(1, f(1))$.
(b) On your graph, draw the tangent line at $x = 1$.
(c) On the same graph, draw a line whose slope is "represented" by $[f(1) - f(.5)]/[1 - .5]$.
(d) Without doing any computations, can you tell which is larger: the average rate of change of f on the interval $[.5, 1]$, or the instantaneous rate of change of f at $x = 1$? Explain your answer.