Math 110 | **BASIC CALCULUS I** | Fall 2007

Class 13

Monday October 1

Tangent Lines, Average and Instantaneous Rates Of Change

Average Velocity versus Instantaneous Velocity

1. Suppose from 8:00 AM to 9:00 AM you travelled 70 miles.
What was your average velocity for this trip?

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2. What was your exact velocity at 8:10 AM?

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3. What if in addition you knew that from 8:09 to 8:15 you travelled 4 miles?
What would you guess your exact velocity was at 8:10 AM?

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4. Suppose we in fact had the following table:

Time	Distance
8:00	0
8:09:00	7
8:09:58	8
8:10:00	8.03
8:15	11
9:00	70

5. Using this data, what would you estimate for the exact velocity at 8:10:00?

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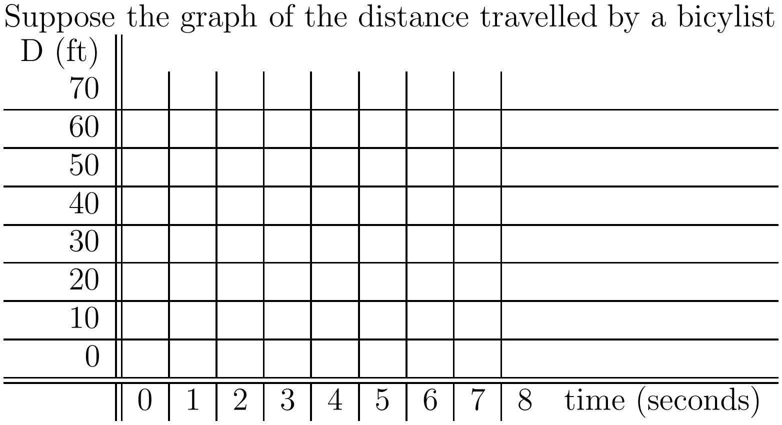
Estimating Rates Of Change From a Graph

Suppose the graph of the distance travelled by a bicylist as a function of time looks as follows.



6. Estimate the velocity of the bicylist at time $t = 5$.

7. Estimate the slope of the graph at time $t = 5$.

8. During which time intervals is the velocity constant?

9. During which time intervals is the velocity increasing?

10. During which time intervals is the velocity decreasing?

11. Estimate how far the bicyclist will travel from during the time interval $[8, 10]$.

Average and Instantaneous Velocity
Previously we have thought about estimating velocity given a table (and a graph) of distance travelled versus time elapsed. Suppose the position of a car at time t seconds is given by (the function) $s(t)$ feet.

Then the **average velocity** of the car between $t = a$ and $t = b$ is given by,

$$v_{ave} = \frac{s(b) - s(a)}{b - a}.$$

The **instantaneous velocity** at $t = a$ is defined by,

$$v_{inst} = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}.$$

Finding the rate of change of a linear function.

The *average rate of change of a function* $y = f(x)$ *on an interval* $[a, b]$ is given by the change in the output divided by the change in the input:

$$\frac{\Delta y}{\Delta x} = \frac{\text{change in output}}{\text{change in input}} = \frac{f(b) - f(a)}{b - a}.$$

$\boxed{\text{EXAMPLE}}$
What is the average rate of change of the function $f(x) = 3x + 2$ on the interval $[4, 10]$?

What is the instantaneous rate of change of the function $f(x) = 3x + 2$ on the interval $[4, 10]$?

Rates and slopes are really the same thing for linear functions.
For a **linear function**, the *instantaneous rate of change* (slope of the line) equals the *average rate of change*.

Tangent Line To A Curve For functions which are not linear, i.e. their graphs are curves the formula for v_{inst} has a graphical interpretation of providing the slope of the **tangent line** to the curve $s(t)$.
$\boxed{\text{DEFINITION: tangent line}}$
The tangent line to the curve $y = f(x)$ at the point $(a, (f(a))$ is described by the equation $y - f(a) = m_{tangent}(x - a)$ where

$$m_{tangent} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided the limit exists.