Basic Calculus I

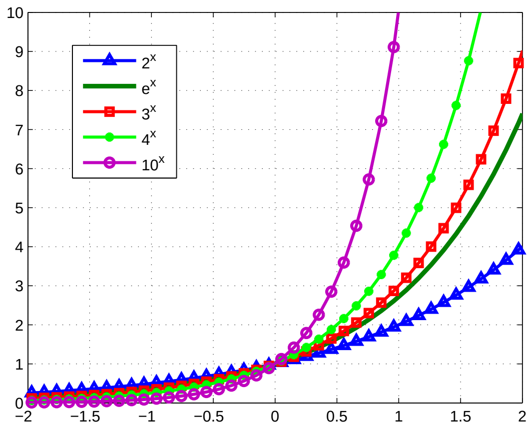
Class 6

Wednesday September 12

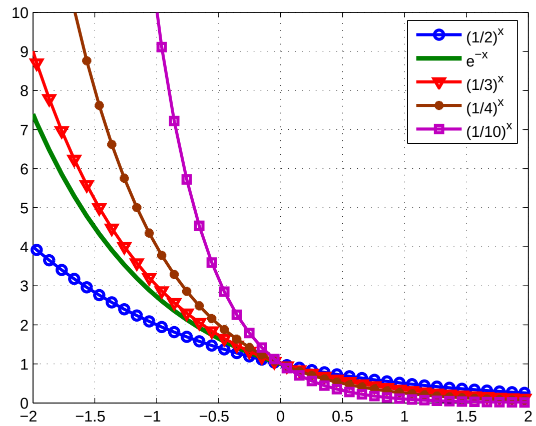
Exponentials and Logarithms

Another Family of Functions

Let's consider the family of functions described by $y = b^x$ where $b > 0$



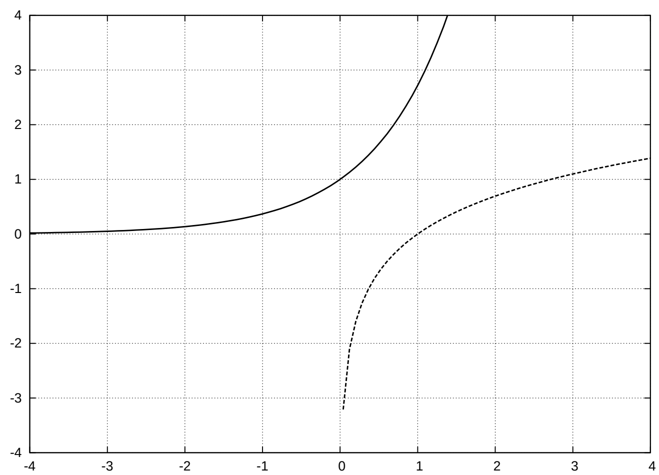
b^x for $b = 2, e, 3, 4,$ and 10



b^x for $b = \frac{1}{2}, \frac{1}{e}, \frac{1}{3}, \frac{1}{4},$ and $\frac{1}{10}$

The number e is approximately 2.718282. The exponential curve e^x is the unique member of the family of curves whose tangent to the curve at the common point $(0, 1)$ has slope exactly equal to 1.

What's so special about e? It is the base of the natural logarithmic function, $\ln(x)$. The function e^x is known as the natural exponential function. The graphs of $y = e^x$ and $y = \ln(x)$ are given below on the same axes.



GroupWork

Observe the graphs carefully and write down as many relationships between these functions as you can deduce.

Algebraic Properties of Logarithms

If $b > 0, b \neq 1, a > 0, c > 0$ and r is any real number, then

(a) $\log_b(ac) = \log_b(a) + \log_b(c)$

(b) $\log_b(a/c) = \log_b(a) - \log_b(c)$

(c) $\log_b(a^r) = r\log_b(a)$

(d) $\log_b(1/c) = -\log_b(c)$

Cancellation Equations

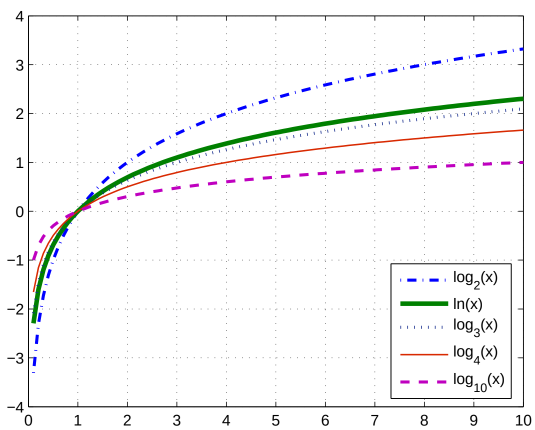
Since $\log_b(x)$ and b^x are inverses of each other (for $b > 0$ and $b \neq 1$) it follows that

$$\begin{aligned} \log_b(b^x) &= x \text{ for all } x \in \mathbb{R} \\ b^{\log_b(x)} &= x \text{ for all } x > 0 \end{aligned}$$

Formula for Change of Logarithmic Base

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$

Logarithmic Family of Curves



Exercise

Anton, Bivens & David, p. 75, Question 1.5.45. Find the error.

$$\begin{aligned} 3 &> 2 \\ 3\log\left(\frac{1}{2}\right) &> 2\log\left(\frac{1}{2}\right) \\ \log\left(\frac{1}{2}\right)^3 &> \log\left(\frac{1}{2}\right)^2 \\ \log\left(\frac{1}{8}\right) &> \log\left(\frac{1}{4}\right) \\ \frac{1}{8} &> \frac{1}{4} \end{aligned}$$