Class 4

Friday September 7

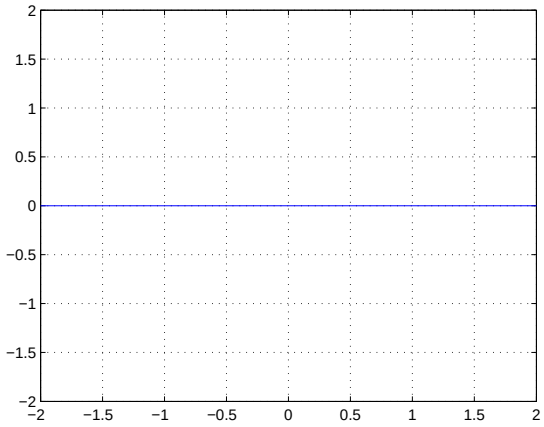
Families of Functions

DEFINITION

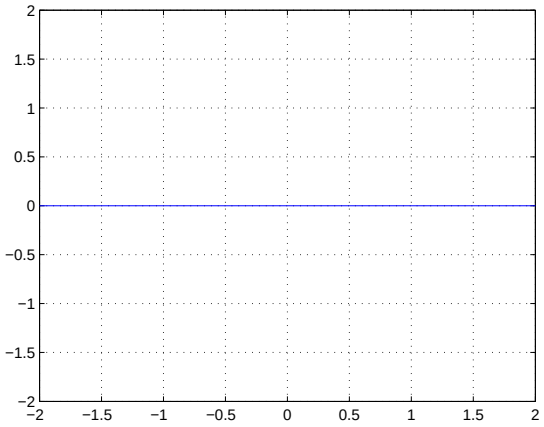
A constant that is varied in order to produce a variety of scenarios or dfferent situations is known as a **parameter**. One example of the use of a parameter is in the definition of a family of functions.

Familes of Curves

Consider the family of curves $y = mx + b$, where both b and m are parameters.



b fixed, m varies (let $b = 1$)

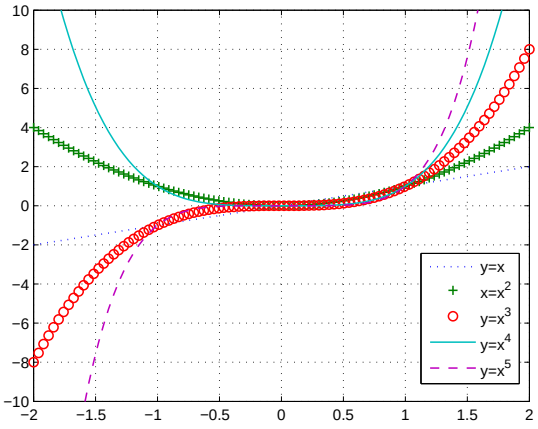


b varies, m fixed (let $m = 1$)

The Power Function

A power function has the form $f(x) = x^p$ where p is a parameter. We'll consider power functions in two cases, CASE 1 where p is positive and CASE 2 where p is negative.

CASE 1 $f(x) = x^p, p > 0$

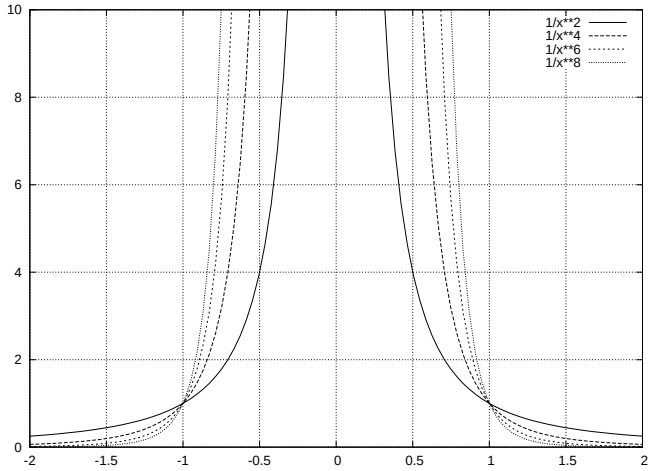


GROUPWORK

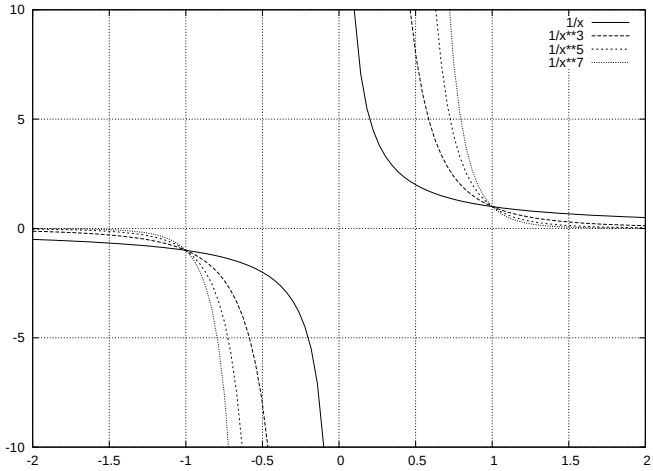
a. Which of the members of the power function family are ODD? Which of them are EVEN? How can you tell, graphically?

b. Are there any points common to all family members? What are they?

CASE 2 $f(x) = x^p, p < 0$
NOTE: This family is written by the textbook as $f(x) = 1/x^n$ where $n > 0$)



$f(x) = 1/x^n, \quad n$ even



$f(x) = 1/x^n, \quad n$ odd

Note, there's no reason why the value of p in a power function $f(x) = x^p$ has to be an integer, it could be any real number. **EXERCISE**

Question: Which of the following functions is a power function? x^π, $x + \pi$, $\dfrac{1}{x+\pi}$, $\sqrt[\pi]{x}$, $\dfrac{1}{x^\pi}$

Answer:

Polynomials
a **polynomial** is a function which is a finite sum of terms of the form cx^n where n is a nonnegative integer. Polynomials are very **tractable** functions because they have domains which consist of all real numbers.

Example
Write down an example of a polynomial. Identify the coefficients of the polynomial as well as its degree.

Rational Functions
A **rational function** is a ratio of two polynomial functions. Rational functions are interesting because their graphs often have vertical or horizontal asymptotes.

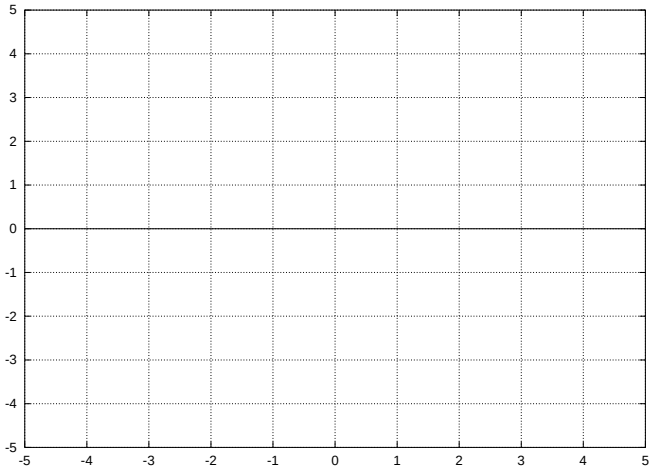
Algebraic Functions
A function which can be obtained by applying a finite number of algebraic operations (addition, subtraction, multiplication, division, and root extraction) to polynomials are called **algebraic functions**.

Trigonometric Functions

Consider the family of functions $A\sin(Bx)$ and $A\cos(Bx)$. Recall, that these are sinusoidal functions with amplitude equal to $|A|$, period $\dfrac{2\pi}{|B|}$ and frequency $\dfrac{|B|}{2\pi}$.

Example

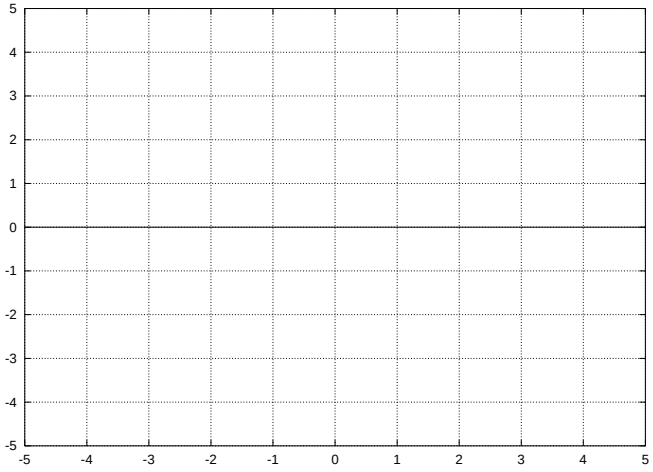
Sketch the graph of $y = 3\sin(2x)$ on the axes below.



The most general form of writing the family of trigonometric functions is as $A\sin(Bx - C)$ and $y = A\cos(Bx - C)$. By re-writing these expressions as $y = A\sin(B\left(x - \dfrac{C}{B}\right))$ and $y = A\cos(B\left(x - \dfrac{C}{B}\right))$ one can see that these graphs have amplitude _____, period _____ and are shifted _____ units to the _____ compared to $A\sin(Bx)$ and $A\cos(Bx)$.

Exercise

Sketch the graph of $y = 3\sin(2x - \pi)$ on the axes below.



Other Resources

Barbara Kaskosz's **Family of Functions** plotter at
`http://www.math.uri.edu/ bkaskosz/flashmo/nov11fin.html` is a useful resource to explore families of functions.