GOAL: This quiz is designed to illuminate your understanding of derivatives, differentiation and continuity. 
TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once.

(a) 5 points. TRUE or FALSE? “There is exactly one function whose derivative equals $x^2 + 3$.”

FALSE

Convert to Math: $M' = x^2 + 3$ where $M$ is unknown

$M(x) = \frac{1}{3}x^3 + 3x + 2 \Rightarrow M' = x^2 + 3$, and $M(x) = \frac{1}{3}x^3 + 3x - 7 \Rightarrow M' = x^2 + 3$.

There are many functions whose derivative equals $x^2 + 3$.

(b) 5 points. TRUE or FALSE? “There is exactly one function which equals the derivative of $x^2 + 3$.”

TRUE

Convert to Math: $(x^2 + 3)' = M'$ where $M$ is unique and unknown

$(x^2 + 3)' = 2x = M'(x)$

There is one function which equals the derivative of $x^2 + 3$ and that function is $2x$.

(c) 5 points. TRUE or FALSE? “If $f(x)$ is continuous at $x = a$, then $f(a)$ exists.”

TRUE

Look at Contrapositive Version:

If $f(a)$ does not exist then $f(x)$ is not continuous at $x = a$. This is true, since to be continuous

$\lim_{x \to a} f(x) = f(a)$, so by definition $f(a)$

of continuity

would exist.

(d) 5 points. TRUE or FALSE? “If $f(a)$ DOES NOT exist, then $f(x)$ is NOT differentiable at $x = a$.”

TRUE

Look at CONTRAPOSITIVE

$f(x)$ is differentiable $\Rightarrow f(a)$ does exist at $x = a$

$f(a)$ exists $\Rightarrow f(x)$ is continuous at $x = a$ $\Rightarrow f(a)$ exists (from part(c)

If $f(a)$ does not exist then this limit cannot be evaluated, so $f'(a)$

will not exist i.e. $f(x)$ is not