Lab Time: **Your Name:** Key

GOAL: This quiz is designed to illuminate your understanding of derivatives, differentiation and continuity. TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once.

(a) *5 points.* **TRUE or FALSE?** "There is exactly one function whose derivative equals $x^2 + 3$."

FALSE

Convert to Math: "$M' = x^2+3$" where M is unknown and unique.

$M(x) = \frac{1}{3}x^3 + 3x + 2 \Rightarrow M' = x^2+3$

and $M(x) = \frac{1}{3}x^3 + 3x - 7 \Rightarrow M' = x^2+3$.

There are many functions whose derivative equals x^2+3.

(b) *5 points.* **TRUE or FALSE?** "There is exactly one function which equals the derivative of $x^2 + 3$."

TRUE

Convert to Math: "$(x^2+3)' = M$" where M is unique and unknown

$(x^2+3)' = 2x = M(x)$

There is one function which equals the derivative of x^2+3 and that function is $2x$.

(c) *5 points.* **TRUE or FALSE?** "If $f(x)$ is continuous at $x = a$, then $f(a)$ exists."

TRUE

① If $f(x)$ is continuous at $x=a$, then $\lim_{x\to a} f(x) = f(a)$, so by definition of continuity $f(a)$ would exist.

② Look at contrapositive version:

$f(a)$ DOES NOT exist $\Rightarrow$ $f(x)$ is NOT CONTINUOUS at $x=a$.

This is true, since to be continuous $\lim_{x\to a} f(x) = f(a)$, if $f(a)$ does not exist $\lim_{x\to a} f(x) \neq f(a)$ and $f(x)$ will not be CONT at $x=a$.

(d) *5 points.* **TRUE or FALSE?** "If $f(a)$ DOES NOT exist, then $f(x)$ is NOT differentiable at $x = a$."

TRUE

① Look at CONTRAPOSITIVE

$f(x)$ is differentiable at $x=a$ $\Rightarrow$ $f(a)$ does exist

$f'(a)$ exists $\Rightarrow$ $f(x)$ is continuous at $x=a$ $\Rightarrow$ $f(a)$ exists (from part (c))

② Look at definition of $f'(a)$

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

If $f(a)$ does not exist then this limit can not be evaluated, so $f'(a)$ will not exist i.e. $f(x)$ is NOT