SHOW ALL YOUR WORK AND EXPLAIN ALL YOUR ANSWERS

Assume that f and g are functions which are unknown but differentiable everywhere.
Given that $f(0) = -1$, $f(1) = -2$, $f'(0) = -1$, $f'(1) = 3$, $g(0) = 3$, $g(1) = 1$, $g'(0) = -1$ and $g'(1) = -2$,

a. *(2 points.)* If $y = 2f(x) - 3g(x)$, evaluate $\left.\frac{dy}{dx}\right|_{x=1}$.

Constant Multiple Rule

$$\frac{dy}{dx} = 2f'(x) - 3g'(x)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 2f'(1) - 3g'(1) = 2\cdot 3 - 3\cdot(-2) = 6+6 = \boxed{12}$$

b. *(2 points.)* Evaluate $\frac{d}{dx}\left[f(x)g(x)\right]\big|_{x=0}$.

Product Rule

$$\frac{d}{dx}(fg) = f'g + fg'$$

$$\left.\frac{d}{dx}(fg)\right|_{x=0} = f'(0)g(0) + f(0)g'(0) = -1\cdot 3 + -1\cdot -1$$
$$= -3 + 1 = \boxed{-2}$$

c. *(2 points.)* Evaluate $\left.\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]\right|_{x=0}$.

Quotient Rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

$$\left.\frac{d}{dx}\left(\frac{f}{g}\right)\right|_{x=0} = \frac{f'(0)g(0) - f(0)g'(0)}{(g(0))^2} = \frac{-1\cdot 3 - (-1)\cdot(-1)}{3^2} = \frac{-3-1}{9} = \boxed{\frac{-4}{9}}$$

d. *(2 points.)* If $p(x) = x^2 f(x)$, evaluate $p'(1)$.

$$p'(x) = 2x\,f(x) + x^2 f'(x)$$ Product Rule

$$p'(1) = 2\cdot 1\cdot f(1) + 1^2 f'(1)$$
$$= 2\cdot -2 + 1\cdot 3 = -4 + 3 = \boxed{-1}$$

e. *(2 points.)* If $q(x) = \frac{x^3}{g(x)}$, evaluate $q'(1)$.

Quotient Rule

$$q' = \frac{3x^2\cdot g(x) - x^3\cdot g'(x)}{g^2}$$

$$q'(1) = \frac{3\cdot 1^2\cdot g(1) - 1^3 g'(1)}{(g(1))^2} = \frac{3\cdot 1 - 1\cdot -2}{1^2} = 3+2 = \boxed{5}$$