SHOW ALL YOUR WORK AND EXPLAIN ALL YOUR ANSWERS

We want to use the information about the function $g(t) = t^3$ to find the equation of the tangent line to $g(t)$ at the point $(2, g(2))$.

t	1.900	1.990	1.999	2.000	2.001	2.010	2.100
$g(t)$	6.859	7.881	7.988	8.000	8.012	8.121	9.261

a. *(4 points.)* Use the following table to produce a sequence of successive approximations in order to find the **exact value** of the slope of the curve $g(t) = t^3$ at the point $(2, g(2))$.

$$\lim_{t \to 2} \frac{g(t) - g(2)}{t-2}$$

$$g'(2) = 12$$

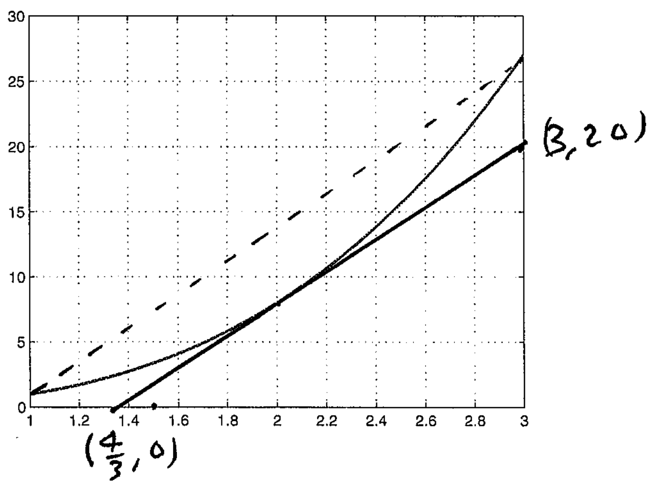
t	$t-2$	$g(t)$	$g(t)-g(2)$	$\frac{g(t)-g(2)}{t-2}$
1.999	-0.001	7.988	-0.012	12
1.99	-0.01	7.881	0.ϕ19	11.9
2.001	+0.001	8.012	+0.012	12
2.0ϕ0	0.0ϕ0	8.121	0.121	12.1
2.1	0.1	9.261	1.261	12.61

b. *(2 points.)* Use your answer from **(a.)** about the exact value of the slope of the curve at $t = 2$ to find the equation of the tangent line to the curve at the point $t = 2$.

$$y - 8 = 12(t-2)$$

$$y = 12t - 24 + 8 = 12t - 16$$

c. *(2 points.)* Sketch the tangent line to the curve at $t = 2$ on the graph below. (Make sure your sketch touches the t-axis and the line $t = 3$.)



d. *(2 points.)* Which is greater, the average rate of change of the function $g(t) = t^3$ on the interval $[1, 3]$ or the instantaneous rate of change of $g(t)$ at the point $t = 2$? EXPLAIN YOUR ANSWER.

$$\frac{g(3) - g(1)}{3-1} \approx \frac{3^3 - 1^3}{2} = \frac{27-1}{2} = \frac{26}{2} = 13$$

$g'(2) \approx 12$. $13 > 12$ so the average rate is greater than instantaneous rate