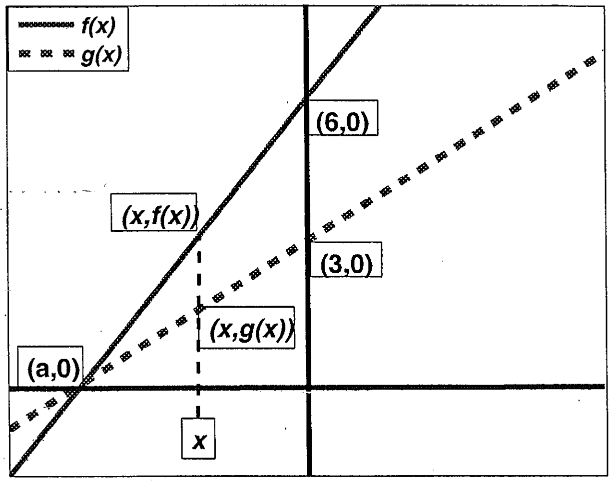
MATH 110: BASIC CALCULUS 1 — Quiz Three: *Limits*
Fall 2007 — *Prof. R. Buckmire*

Lab Time: **Your Name:** BUCKMIRE

GOAL: This quiz is designed to illuminate your understanding of limits, visually, computationally and conceptually.



1. (*20 points TOTAL.*) Consider the two *unknown* **linear** functions $f(x)$ and $g(x)$ graphed in the figure above. The two lines have different y-intercepts but share the same x-intercept.

Evaluate the following limits. In each case, EXPLAIN YOUR ANSWER. If you do not think the limit exists, explain why.

(a) (*3 points.*) $\boxed{\lim_{x\to 0^-} f(x) = 6}$

Since $f(x)$ is a linear function; ie. a polynomial $\lim_{x\to 0^-} f(x) = f(0) = 6$. From the graph, as x approaches 0 from the left, output values of $f(x)$ approach 6.

(b) (*3 points.*) $\boxed{\lim_{x\to a^+} g(x) = 0}$

Since $g(x)$ is ALSO a polynomial $\lim_{x\to a^+} g(x) = g(a) = 0$. From the graph, as x approaches a from the right, output values of $f(x)$ approach 0.

(c) (*3 points.*) $\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)} = \frac{6}{3} = \boxed{2}$

limit of a quotient is quotient of the limits

(d) (*3 points.*) $\lim_{x\to 0} f(x)g(x) = \lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x) = 6 \cdot 3 = \boxed{18}$

limit of a product is the product of the limits

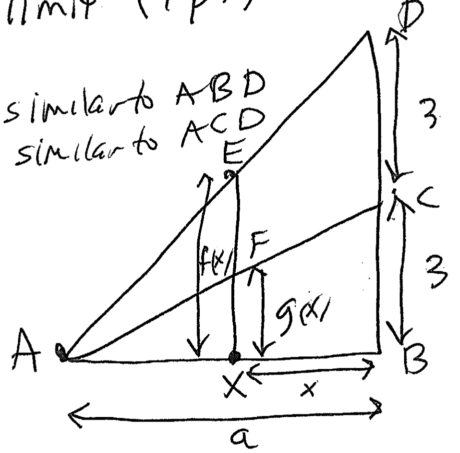
(e) (*3 points.*) $\lim_{x\to 0} 4f(x) - 5(g(x) = 4\lim_{x\to 0} f(x) - 5\lim_{x\to 0} g(x) = 4\cdot 6 - 5\cdot 3$
$= 24 - 15$
$= \boxed{9}$

$\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$ AND limit of a difference is difference of limits

(f) (*5 points.*) $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ [HINT: Use similar triangles to obtain a simple algebraic relationship between $f(x)$ and $g(x)$]

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$$ indeterminate limit (1pt)

Using similar triangles AEX is similar to ABD and AFX is similar to ACD



$$\frac{f(x)}{6} = \cancel{\frac{g(x)}{3}} \; \frac{a-x}{a} = \frac{g(x)}{3}$$

$$\Rightarrow \frac{f(x)}{6} = \frac{g(x)}{3} \Rightarrow f(x) = 2g(x)$$
$$\frac{f(x)}{g(x)} = 2$$

$$\boxed{\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} 2 = 2}$$

Alternative solution:

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{m_f x + 6}{m_g x + 3} = \lim_{x\to\infty} \frac{m_f}{m_g} = \boxed{2}$$

m_f = slope of $f(x) = \frac{6}{-a}$

m_g = slope of $g(x) = \frac{3}{-a}$

$$\frac{m_f}{m_g} = \frac{\frac{6}{-a}}{\frac{3}{-a}} = \frac{6}{3}$$

BONUS (*5 points.*) Evaluate $\lim_{x\to a} \frac{f(x)}{g(x)}$. Describe carefully what techniques you use to find the value of the limit, if it exists, or explain why the limit doesn't exist.

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)} = \frac{0}{0}$$ Indeterminate form, but we know $f(x) = 2g(x)$ for EVERY x

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{6}{3} = \boxed{2}$$

In this case one ~~must~~ can use similar triangles to obtain the relationship that $f(x) = 2g(x)$

Another way is to compute the slopes of $f(x)$ and $g(x)$

Thus $f(x) = \left(\frac{6}{-a}\right)x + 6$, $g(x) = \frac{3}{-a}x + 3$

$$\lim_{x\to a} \frac{\frac{6}{-a}x + 6}{\frac{3}{-a}x + 3} = \lim_{x\to a} \frac{6x - 6a}{3x - 3a} = \lim_{x\to a} \frac{2(3x-3a)}{(3x-3a)} = \lim_{x\to a} 2 = \boxed{2}$$