SHOW ALL YOUR WORK AND EXPLAIN ALL YOUR ANSWERS

Consider the two functions \( f(x) = \sqrt{x}, x \geq 0 \) and \( g(x) = x^4, -\infty < x < \infty \).

(a) (3 points) Is \( f(x) \) invertible? If not, why not? If it is invertible, say why. Is \( f(x) \) even, odd or neither? EXPLAIN YOUR ANSWERS.

\[
\begin{align*}
\text{Yes, } f(x) &= \sqrt{x}, x \geq 0 \text{ is invertible since it passes the horizontal line test.} \\
\text{No, } f(-x) &= \text{ undefined when } x \leq 0
\end{align*}
\]

(b) (3 points) Is \( g(x) \) invertible? If not, why not? If it is invertible, say why. Is \( g(x) \) even, odd or neither? EXPLAIN YOUR ANSWERS.

\[
\begin{align*}
\text{No, } g(\pm x) &= (-x)^4 = x^4 = g(x) \text{ it does not pass the horizontal line test!} \\
\text{Yes, } g(x) \text{ is even. } g(-x) &= (-x)^4 = x^4 = g(x).
\end{align*}
\]

(c) (2 points) Compute \((g \circ f)(x)\) and \((f \circ g)(x)\) and give their domains. Are these functions different?

\[
\begin{align*}
(g \circ f)(x) &= g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^4 = x^2, \quad x \geq 0 \\
(f \circ g)(x) &= f(g(x)) = f(x^4) = \sqrt{x^4} = x^2, \quad -\infty < x \leq 0
\end{align*}
\]

(d) (2 points) Sketch the graphs of \((g \circ f)(x)\) and \((f \circ g)(x)\) on the axes below.

- Graph of \( g \circ f \): **INVERTIBLE**
- Graph of \( f \circ g \): **NOT INVERTIBLE**