Math 110 Fall 2007 BONUS Quiz 4

SHOW ALL YOUR WORK AND EXPLAIN EVERY ANSWER

Consider the initial value problem (IVP) below

$$\begin{aligned} C'(t) &= 2t\cdot(C(t))^2 \\ C(1) &= -1. \end{aligned}$$

(a) *(6 points)* Use Euler's method with a time step of $\Delta t = 1/2$ to fill in the table below.

t	$C(t)$	Δt	$C'(t)$	ΔC
1	-1	1/2	2	1
1 1/2	0	1/2	0	0
2	0	1/2	XXXXXXXX XXXXXXXX XXXXXXXX	XXXXXXXX XXXXXXXX XXXXXXXX

(b) *(2 points)* Consider the function $C(t) = -t^{-2}$. Is this the solution to the given initial value problem?

Check IC $C(1) = -(1)^{-2} = -\frac{1}{1^2} = -1$ ✓

Check DE $\frac{dC}{dt} = \frac{d}{dt}(-t^{-2}) = -(-2t^{-3}) = 2t^{-3} = 2t\cdot\frac{1}{t^4} = 2t\left(-\frac{1}{t^2}\right)^2 = 2tC^2$ ✓

(c) *(2 points)* Determine whether the approximation computed in part (a) for $C(2)$ is an over-estimate or under-estimate.

$C(2) = -2^{-2} = -\frac{1}{4} < 0 \Rightarrow$ so, Euler's is an OVERESTIMATE

OR

$C'' = 2\cdot C^2 + 2t\cdot 2C\cdot C'$

$= 2C^2 + 2t2C\cdot 2tC^2$

$C''(1) = 2\cdot(-1)^2 + 2\cdot 1\cdot 2\cdot(-1)\cdot 2\cdot(1)\cdot(-1)^2$

$= 2\cdot 1 + 2\cdot -2\cdot 1\cdot 2$

$= 2 - 8 = -6 < 0$ C is CONCAVE DOWN, so Euler's is OVERESTIMATE