Math 110 Fall 2007 **BONUS** Quiz 1

SHOW ALL YOUR WORK

1. *(10 points total)* Recall that the limit definition of the derivative function is $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

(a) *(8 points)* Given $f(x) = \sqrt{x}$, use the limit definition of the derivative to obtain $f'(x)$ algebraically.

STEP 1. Simplify DQ

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{(\sqrt{x+h} - \sqrt{x}) \cdot (\sqrt{x+h} + \sqrt{x})}{h \cdot (\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

STEP 2 Take limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$f(x) = \sqrt{x} = x^{1/2}, \quad f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-1/2}$$

(b) *(2 points)* Is the domain of $f'(x)$ different from the domain of $f(x)$? EXPLAIN YOUR ANSWER.

Yes! Domain of $f(x) = \sqrt{x}$ is $[0, \infty)$

Domain of $f'(x) = \frac{1}{2\sqrt{x}}$ is $(0, \infty)$