Lab 12: Curve Sketching

Objectives:

1. To find intervals on which a function is increasing or decreasing
2. To find intervals on which a function is positive or negative
3. To find intervals on which a function is concave up or concave down
4. To classify the asymptotic behavior of the function
5. To synthesize the above information to produce a sketch a graph of a function

Introduction

In this lab, we want to apply all the techniques we have learned in this section of the course to sketch graphs for four different functions. Turn them in during the lab without using any graphing software. Then you may use DERIVE or your TI-83 (graphing calculator) to get the correct graphs, compare them with your answers, and correct your work.

For the following functions, look for extrema, intercepts, symmetry, and asymptotes as necessary. Identify intervals on which the function is increasing, decreasing, concave up and concave down. Then use this information to produce a sketch of the function, which you should compare to the graph your calculator or `Derive` produces.

On the following page is a summary of graphing techniques which you may find useful.

Summary of Graphing Techniques

This is a discription of how one might organize the information needed to sketch the graph of a function. Try this method a few times. Then develop your own systematic method (feel free to adopt this method if you wish). Remember, not all steps are needed in every situation. If a step is unimportant to the context of your problem or if a step is simply too difficult, skipping that step is fine.

Step #1: General and Extreme Behavior

A. Determine if $f(x)$ is even or odd or neither.

B. Determine if $f(x)$ is periodic and if so, what is its period.

C. Determine the behavior of $f(x)$ as $x \to \pm\infty$.

D. Find any asymptotes, vertical, horizontal (and slant, for the adventurous).

E. Find the y-intercept.

Step #2: $(+, -)$: Positive, Negative, Zeros.

A. Find the critical points for the positive or negative analysis, i.e. the endpoints, the zeros of $f(x)$ and the x-values which make the function $f(x)$ undefined.

B. Plot the critical points on the $(+, -)$ number line and check the intervals between the critical points to determine where the function is positive and where it is negative.

Remark: Sometimes this step is more work than it is worth. Do this step only if it is easy. You can find approximations of the roots using trial and error or Newton's method, depending on how accurate you need to be.

Step #3: $(\uparrow, \downarrow)$: Increasing, Decreasing, Relative Extrema.

A. Find the critical points for increasing and decreasing i.e. the endpoints, the zeros of $f'(x)$ and the x-values which make the derivative function undefined.

B. Plot the critical points on the $(\uparrow, \downarrow)$ number line and check the intervals between the critical points to determine where the derivative function is positive and where it is negative. This tells us where the original function is increasing, decreasing and where it has local extrema.

Remark: You may need to approximate the roots using trial and error, the bisection method or Newton's method, depending, again, on how accurate you need to be.

Step #4: $(\cup, \cap)$: Concavity, Inflection Points.

A. Find the critical points for concavity, i.e. the endpoints, the zeros of $f''(x)$ and the x-values which make the second derivative function undefined.

B. Plot the critical points on the $(\cup, \cap)$ number line and check the intervals between the critical points to determine where the second derivative function is positive and where it is negative. This tells us where the original function is concave up, concave down and where it has points of inflection.

Remark: Again, finding approximations for the roots may involve using trial and error, the bisection method or Newton's method.

Step #5: Plotting.

A. Plot all critical points and asymptotes.

B. Use the $(+, -)$, $(\uparrow, \downarrow)$ and $(\cup, \cap)$ number lines to draw the graph, plotting additional points if greater accuracy is desired.

§1. $f(x) = 3x^4 - 4x^3$

1. Find the roots of the function.

2. Identify the intervals on which the function is positive and negative.

3. Find the critical points of the function.

4. Identify the intervals on which the function is increasing and decreasing.

5. Find the inflection points of the function.

6. Identify the intervals on which the function is concave up and concave down.

7. Determine whether the function has any horizontal asymptotes.

8. Determine whether the function is undefined at any points.

9. Check to see if these points are vertical asymptotes

10. Sketch the function below.

§2. $g(x) = \dfrac{x^2}{x^2 - x - 2}$

1. Find the roots of the function.

2. Identify the intervals on which the function is positive and negative.

3. Find the critical points of the function.

4. Identify the intervals on which the function is increasing and decreasing.

5. Find the inflection points of the function.

6. Identify the intervals on which the function is concave up and concave down.

7. Determine whether the function has any horizontal asymptotes.

8. Determine whether the function is undefined at any points.

9. Check to see if these points are vertical asymptotes

10. Sketch the function below.

§3. $i(x) = \dfrac{x^2 - 9}{2x - 4}$

1. Find the roots of the function.

2. Identify the intervals on which the function is positive and negative.

3. Find the critical points of the function.

4. Identify the intervals on which the function is increasing and decreasing.

5. Find the inflection points of the function.

6. Identify the intervals on which the function is concave up and concave down.

7. Determine whether the function has any horizontal asymptotes.

8. Determine whether the function is undefined at any points.

9. Check to see if these points are vertical asymptotes

10. Sketch the function below.

Write-Up

Your lab write-up should consist of one neat copy of this lab, handed in one week from today on **November 29, 2007**. One signed copy of the Grade Allocation for Lab Assignments should be attached as a cover sheet.