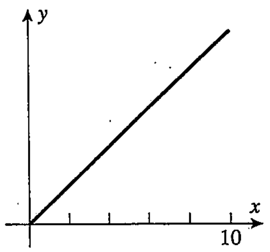
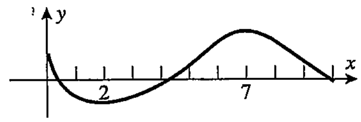
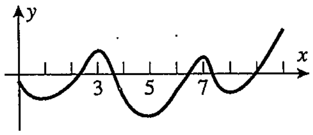
EXERCISE SET 5.4

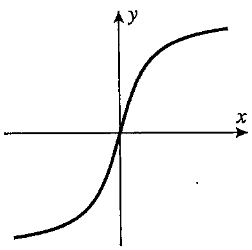
1. relative maxima at $x = 2, 6$; absolute maximum at $x = 6$; relative and absolute minima at $x = 0, 4$

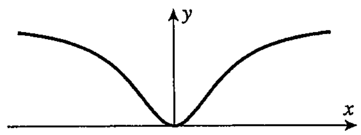
2. relative maximum at $x = 3$; absolute maximum at $x = 7$; relative minima at $x = 1, 5$; absolute minima at $x = 1, 5$

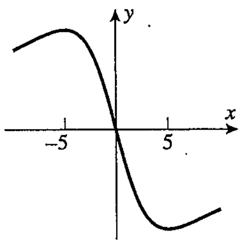
3. (a) 

(b) 

(c) 

4. (a) 

(b) 

(c) 

5. $x = 1$ is a point of discontinuity of f.

6. Since f is monotonically increasing on $(0, 1)$, one might expect a minimum at $x = 0$ and a maximum at $x = 1$. But both points are discontinuities of f.

7. $f'(x) = 8x - 12$, $f'(x) = 0$ when $x = 3/2$; $f(1) = 2$, $f(3/2) = 1$, $f(2) = 2$ so the maximum value is 2 at $x = 1, 2$ and the minimum value is 1 at $x = 3/2$.

8. $f'(x) = 8 - 2x$, $f'(x) = 0$ when $x = 4$; $f(0) = 0$, $f(4) = 16$, $f(6) = 12$ so the maximum value is 16 at $x = 4$ and the minimum value is 0 at $x = 0$.

9. $f'(x) = 3(x-2)^2, f'(x) = 0$ when $x = 2$; $f(1) = -1, f(2) = 0, f(4) = 8$ so the minimum is -1 at $x = 1$ and the maximum is 8 at $x = 4$.

10. $f'(x) = 6x^2 + 6x - 12$, $f'(x) = 0$ when $x = -2, 1$; $f(-3) = 9$, $f(-2) = 20$, $f(1) = -7$, $f(2) = 4$, so the minimum is -7 at $x = 1$ and the maximum is 20 at $x = -2$.

11. $f'(x) = 3/(4x^2+1)^{3/2}$, no critical points; $f(-1) = -3/\sqrt{5}$, $f(1) = 3/\sqrt{5}$ so the maximum value is $3/\sqrt{5}$ at $x = 1$ and the minimum value is $-3/\sqrt{5}$ at $x = -1$.

12. $f'(x) = \dfrac{2(2x+1)}{3(x^2+x)^{1/3}}$, $f'(x) = 0$ when $x = -1/2$ and $f'(x)$ does not exist when $x = -1, 0$; $f(-2) = 2^{2/3}$, $f(-1) = 0$, $f(-1/2) = 4^{-2/3}$, $f(0) = 0$, $f(3) = 12^{2/3}$ so the maximum value is $12^{2/3}$ at $x = 3$ and the minimum value is 0 at $x = -1, 0$.

13. $f'(x) = 1 - 2\cos x$, $f'(x) = 0$ when $x = \pi/3$; then $f(-\pi/4) = -\pi/4 + \sqrt{2}$; $f(\pi/3) = \pi/3 - \sqrt{3}$; $f(\pi/2) = \pi/2 - 2$, so f has a minimum of $\pi/3 - \sqrt{3}$ at $x = \pi/3$ and a maximum of $-\pi/4 + \sqrt{2}$ at $x = -\pi/4$.

14. $f'(x) = \cos x + \sin x$, $f'(x) = 0$ for x in $(0, \pi)$ when $x = 3\pi/4$; $f(0) = -1$, $f(3\pi/4) = \sqrt{2}$, $f(\pi) = 1$ so the maximum value is $\sqrt{2}$ at $x = 3\pi/4$ and the minimum value is -1 at $x = 0$.

15. $f(x) = 1 + |9 - x^2| = \begin{cases} 10 - x^2, & |x| \le 3 \\ -8 + x^2, & |x| > 3 \end{cases}$, $f'(x) = \begin{cases} -2x, & |x| < 3 \\ 2x, & |x| > 3 \end{cases}$ thus $f'(x) = 0$ when $x = 0$, $f'(x)$ does not exist for x in $(-5, 1)$ when $x = -3$ because $\lim\limits_{x\to -3^-} f'(x) \ne \lim\limits_{x\to -3^+} f'(x)$ (see Theorem preceding Exercise 61, Section 3.3); $f(-5) = 17$, $f(-3) = 1$, $f(0) = 10$, $f(1) = 9$ so the maximum value is 17 at $x = -5$ and the minimum value is 1 at $x = -3$.

16. $f(x) = |6 - 4x| = \begin{cases} 6 - 4x, & x \le 3/2 \\ -6 + 4x, & x > 3/2 \end{cases}$, $f'(x) = \begin{cases} -4, & x < 3/2 \\ 4, & x > 3/2 \end{cases}$, $f'(x)$ does not exist when $x = 3/2$ thus $3/2$ is the only critical point in $(-3, 3)$; $f(-3) = 18$, $f(3/2) = 0$, $f(3) = 6$ so the maximum value is 18 at $x = -3$ and the minimum value is 0 at $x = 3/2$.

17. $f'(x) = 2x - 1$, $f'(x) = 0$ when $x = 1/2$; $f(1/2) = -9/4$ and $\lim\limits_{x\to\pm\infty} f(x) = +\infty$. Thus f has a minimum of $-9/4$ at $x = 1/2$ and no maximum.

18. $f'(x) = -4(x+1)$; critical point $x = -1$. Maximum value $f(-1) = 5$, no minimum.

19. $f'(x) = 12x^2(1 - x)$; critical points $x = 0, 1$. Maximum value $f(1) = 1$, no minimum because $\lim\limits_{x\to+\infty} f(x) = -\infty$.

20. $f'(x) = 4(x^3 + 1)$; critical point $x = -1$. Minimum value $f(-1) = -3$, no maximum.

21. No maximum or minimum because $\lim\limits_{x\to+\infty} f(x) = +\infty$ and $\lim\limits_{x\to-\infty} f(x) = -\infty$.

22. No maximum or minimum because $\lim\limits_{x\to+\infty} f(x) = +\infty$ and $\lim\limits_{x\to-\infty} f(x) = -\infty$.

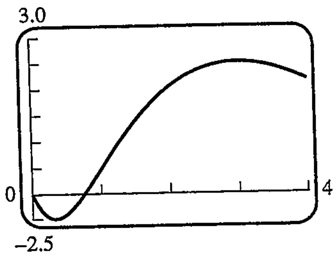
23. $\lim\limits_{x\to -1^-} f(x) = -\infty$, so there is no absolute minimum on the interval;
$f'(x) = 0$ at $x \approx -2.414213562$, for which $y \approx -4.828427125$. Also $f(-5) = -13/2$, so the absolute maximum of f on the interval is $y \approx -4.828427125$ taken at $x \approx -2.414213562$.

24. $\lim\limits_{x\to -1^+} f(x) = -\infty$, so there is no absolute minimum on the interval.
$f'(x) = 3/(x+1)^2 > 0$, so f is increasing on the interval $(-1, 5]$ and the maximum must occur at the endpoint $x = 5$ where $f(5) = 1/2$.

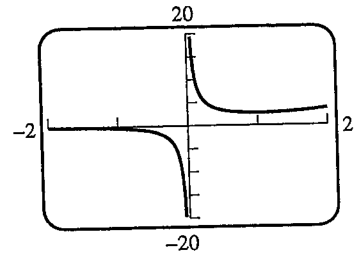
35. $f'(x) = -\dfrac{3x^2 - 10x + 3}{x^2 + 1}$, $f'(x) = 0$ when $x = \dfrac{1}{3}, 3$.

Then $f(0) = 0$, $f\left(\dfrac{1}{3}\right) = 5\ln\left(\dfrac{10}{9}\right) - 1$,

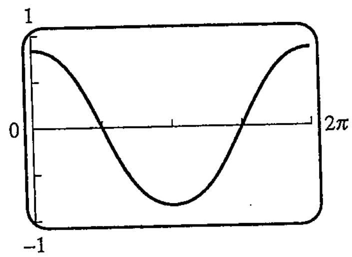
$f(3) = 5\ln 10 - 9$, $f(4) = 5\ln 17 - 12$ and thus f has an absolute minimum of $5(\ln 10 - \ln 9) - 1$ at $x = 1/3$ and an absolute maximum of $5\ln 10 - 9$ at $x = 3$.



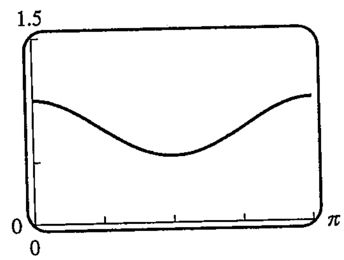
36. $f'(x) = (x^2 + 2x - 1)e^x$, $f'(x) = 0$ at $x = -2 + \sqrt{2}$ and $x = -1 - \sqrt{2}$ (discard), $f(-1+\sqrt{2}) = (2 - 2\sqrt{2})e^{(-1+\sqrt{2})} \approx -1.25$, absolute maximum at $x = 2$, $f(2) = 3e^2 \approx 22.17$, absolute minimum at $x = -1 + \sqrt{2}$



37. $f'(x) = -[\cos(\cos x)]\sin x$; $f'(x) = 0$ if $\sin x = 0$ or if $\cos(\cos x) = 0$. If $\sin x = 0$, then $x = \pi$ is the critical point in $(0, 2\pi)$; $\cos(\cos x) = 0$ has no solutions because $-1 \le \cos x \le 1$. Thus $f(0) = \sin(1)$, $f(\pi) = \sin(-1) = -\sin(1)$, and $f(2\pi) = \sin(1)$ so the maximum value is $\sin(1) \approx 0.84147$ and the minimum value is $-\sin(1) \approx -0.84147$.



38. $f'(x) = -[\sin(\sin x)]\cos x$; $f'(x) = 0$ if $\cos x = 0$ or if $\sin(\sin x) = 0$. If $\cos x = 0$, then $x = \pi/2$ is the critical point in $(0, \pi)$; $\sin(\sin x) = 0$ if $\sin x = 0$, which gives no critical points in $(0, \pi)$. Thus $f(0) = 1$, $f(\pi/2) = \cos(1)$, and $f(\pi) = 1$ so the maximum value is 1 and the minimum value is $\cos(1) \approx 0.54030$.



39. $f'(x) = \begin{cases} 4, & x < 1 \\ 2x - 5, & x > 1 \end{cases}$ so $f'(x) = 0$ when $x = 5/2$, and $f'(x)$ does not exist when $x = 1$ because $\lim_{x\to 1^-} f'(x) \ne \lim_{x\to 1^+} f'(x)$ (see Theorem preceding Exercise 61, Section 3.3); $f(1/2) = 0$, $f(1) = 2$, $f(5/2) = -1/4$, $f(7/2) = 3/4$ so the maximum value is 2 and the minimum value is $-1/4$.

40. $f'(x) = 2x + p$ which exists throughout the interval $(0, 2)$ for all values of p so $f'(1) = 0$ because $f(1)$ is an extreme value, thus $2 + p = 0$, $p = -2$. $f(1) = 3$ so $1^2 + (-2)(1) + q = 3$, $q = 4$ thus $f(x) = x^2 - 2x + 4$ and $f(0) = 4$, $f(2) = 4$ so $f(1)$ is the minimum value.

41. The period of $f(x)$ is 2π, so check $f(0) = 3$, $f(2\pi) = 3$ and the critical points.
$f'(x) = -2\sin x - 2\sin 2x = -2\sin x(1 + 2\cos x) = 0$ on $[0, 2\pi]$ at $x = 0, \pi, 2\pi$ and $x = 2\pi/3, 4\pi/3$. Check $f(\pi) = -1$, $f(2\pi/3) = -3/2$, $f(4\pi/3) = -3/2$.
Thus f has an absolute maximum on $(-\infty, +\infty)$ of 3 at $x = 2k\pi, k = 0, \pm 1, \pm 2, \ldots$ and an absolute minimum of $-3/2$ at $x = 2k\pi \pm 2\pi/3, k = 0, \pm 1, \pm 2, \ldots$.