EXERCISE SET 3.6

1. $(f \circ g)'(x) = f'(g(x))g'(x)$ so $(f \circ g)'(0) = f'(g(0))g'(0) = f'(0)(3) = (2)(3) = 6$

2. $(f \circ g)'(2) = f'(g(2))g'(2) = 5(-3) = -15$

3. (a) $(f \circ g)(x) = f(g(x)) = (2x-3)^5$ and $(f \circ g)'(x) = f'(g(x))g'(x) = 5(2x-3)^4(2) = 10(2x-3)^4$
   (b) $(g \circ f)(x) = g(f(x)) = 2x^5 - 3$ and $(g \circ f)'(x) = g'(f(x))f'(x) = 2(5x^4) = 10x^4$

4. (a) $(f \circ g)(x) = 5\sqrt{4+\cos x}$ and $(f \circ g)'(x) = f'(g(x))g'(x) = \dfrac{5}{2\sqrt{4+\cos x}}(-\sin x)$

   (b) $(g \circ f)(x) = 4 + \cos(5\sqrt{x})$ and $(g \circ f)'(x) = g'(f(x))f'(x) = -\sin(5\sqrt{x})\dfrac{5}{2\sqrt{x}}$

5. (a) $F'(x) = f'(g(x))g'(x) = f'(g(3))g'(3) = -1(7) = -7$
   (b) $G'(x) = g'(f(x))f'(x) = g'(f(3))f'(3) = 4(-2) = -8$

6. (a) $F'(x) = f'(g(x))g'(x)$, $F'(-1) = f'(g(-1))g'(-1) = f'(2)(-3) = (4)(-3) = -12$
   (b) $G'(x) = g'(f(x))f'(x)$, $G'(-1) = g'(f(-1))f'(-1) = -5(3) = -15$

7. $f'(x) = 37(x^3+2x)^{36}\dfrac{d}{dx}(x^3+2x) = 37(x^3+2x)^{36}(3x^2+2)$

8. $f'(x) = 6(3x^2+2x-1)^5\dfrac{d}{dx}(3x^2+2x-1) = 6(3x^2+2x-1)^5(6x+2) = 12(3x^2+2x-1)^5(3x+1)$

9. $f'(x) = -2\left(x^3 - \dfrac{7}{x}\right)^{-3}\dfrac{d}{dx}\left(x^3 - \dfrac{7}{x}\right) = -2\left(x^3 - \dfrac{7}{x}\right)^{-3}\left(3x^2 + \dfrac{7}{x^2}\right)$

10. $f(x) = (x^5 - x + 1)^{-9}$,
    $f'(x) = -9(x^5 - x + 1)^{-10}\dfrac{d}{dx}(x^5 - x + 1) = -9(x^5 - x + 1)^{-10}(5x^4 - 1) = -\dfrac{9(5x^4-1)}{(x^5 - x + 1)^{10}}$

11. $f(x) = 4(3x^2 - 2x + 1)^{-3}$,
    $f'(x) = -12(3x^2 - 2x + 1)^{-4}\dfrac{d}{dx}(3x^2 - 2x + 1) = -12(3x^2 - 2x + 1)^{-4}(6x - 2) = \dfrac{24(1-3x)}{(3x^2 - 2x + 1)^4}$

12. $f'(x) = \dfrac{1}{2\sqrt{x^3 - 2x + 5}}\dfrac{d}{dx}(x^3 - 2x + 5) = \dfrac{3x^2 - 2}{2\sqrt{x^3 - 2x + 5}}$

13. $f'(x) = \dfrac{1}{2\sqrt{4+3\sqrt{x}}}\dfrac{d}{dx}(4 + 3\sqrt{x}) = \dfrac{3}{4\sqrt{x}\sqrt{4+3\sqrt{x}}}$

14. $f'(x) = \dfrac{1}{2\sqrt{\sqrt{x}}}\dfrac{d}{dx}(\sqrt{x}) = \dfrac{1}{2x^{1/4}}\dfrac{1}{2\sqrt{x}} = \dfrac{1}{4x^{3/4}}$

15. $f'(x) = \cos(1/x^2)\dfrac{d}{dx}(1/x^2) = -\dfrac{2}{x^3}\cos(1/x^2)$

16. $f'(x) = (\sec^2\sqrt{x})\dfrac{d}{dx}\sqrt{x} = (\sec^2\sqrt{x})\dfrac{1}{2\sqrt{x}}$

17. $f'(x) = 20\cos^4 x\dfrac{d}{dx}(\cos x) = 20\cos^4 x(-\sin x) = -20\cos^4 x\sin x$

30. $\dfrac{dy}{dx} = \dfrac{\sec(3x+1)\cos x - 3\sin x \sec(3x+1)\tan(3x+1)}{\sec^2(3x+1)} = \cos x\cos(3x+1) - 3\sin x\sin(3x+1)$

31. $\dfrac{dy}{dx} = -\sin(\cos x)\dfrac{d}{dx}(\cos x) = -\sin(\cos x)(-\sin x) = \sin(\cos x)\sin x$

32. $\dfrac{dy}{dx} = \cos(\tan 3x)\dfrac{d}{dx}(\tan 3x) = 3\sec^2 3x\cos(\tan 3x)$

33. $\dfrac{dy}{dx} = 3\cos^2(\sin 2x)\dfrac{d}{dx}[\cos(\sin 2x)] = 3\cos^2(\sin 2x)[-\sin(\sin 2x)]\dfrac{d}{dx}(\sin 2x)$

$= -6\cos^2(\sin 2x)\sin(\sin 2x)\cos 2x$

34. $\dfrac{dy}{dx} = \dfrac{(1-\cot x^2)(-2x\csc x^2\cot x^2) - (1+\csc x^2)(2x\csc^2 x^2)}{(1-\cot x^2)^2} = -2x\csc x^2\,\dfrac{1+\cot x^2+\csc x^2}{(1-\cot x^2)^2}$

35. $\dfrac{dy}{dx} = (5x+8)^7\dfrac{d}{dx}(1-\sqrt{x})^6 + (1-\sqrt{x})^6\dfrac{d}{dx}(5x+8)^7$

$= 6(5x+8)^7(1-\sqrt{x})^5\dfrac{-1}{2\sqrt{x}} + 7\cdot 5(1-\sqrt{x})^6(5x+8)^6$

$= \dfrac{-3}{\sqrt{x}}(5x+8)^7(1-\sqrt{x})^5 + 35(1-\sqrt{x})^6(5x+8)^6$

36. $\dfrac{dy}{dx} = (x^2+x)^5\dfrac{d}{dx}\sin^8 x + (\sin^8 x)\dfrac{d}{dx}(x^2+x)^5 = 8(x^2+x)^5\sin^7 x\cos x + 5(\sin^8 x)(x^2+x)^4(2x+1)$

37. $\dfrac{dy}{dx} = 3\left[\dfrac{x-5}{2x+1}\right]^2\dfrac{d}{dx}\left[\dfrac{x-5}{2x+1}\right] = 3\left[\dfrac{x-5}{2x+1}\right]^2\cdot\dfrac{11}{(2x+1)^2} = \dfrac{33(x-5)^2}{(2x+1)^4}$

38. $\dfrac{dy}{dx} = 17\left(\dfrac{1+x^2}{1-x^2}\right)^{16}\dfrac{d}{dx}\left(\dfrac{1+x^2}{1-x^2}\right) = 17\left(\dfrac{1+x^2}{1-x^2}\right)^{16}\dfrac{(1-x^2)(2x)-(1+x^2)(-2x)}{(1-x^2)^2}$

$= 17\left(\dfrac{1+x^2}{1-x^2}\right)^{16}\dfrac{4x}{(1-x^2)^2} = \dfrac{68x(1+x^2)^{16}}{(1-x^2)^{18}}$

39. $\dfrac{dy}{dx} = \dfrac{(4x^2-1)^8(3)(2x+3)^2(2) - (2x+3)^3(8)(4x^2-1)^7(8x)}{(4x^2-1)^{16}}$

$= \dfrac{2(2x+3)^2(4x^2-1)^7[3(4x^2-1) - 32x(2x+3)]}{(4x^2-1)^{16}} = -\dfrac{2(2x+3)^2(52x^2+96x+3)}{(4x^2-1)^9}$

40. $\dfrac{dy}{dx} = 12[1+\sin^3(x^5)]^{11}\dfrac{d}{dx}[1+\sin^3(x^5)]$

$= 12[1+\sin^3(x^5)]^{11}3\sin^2(x^5)\dfrac{d}{dx}\sin(x^5) = 180x^4[1+\sin^3(x^5)]^{11}\sin^2(x^5)\cos(x^5)$

41. $\dfrac{dy}{dx} = 5\left[x\sin 2x + \tan^4(x^7)\right]^4\dfrac{d}{dx}\left[x\sin 2x\tan^4(x^7)\right]$

$= 5\left[x\sin 2x + \tan^4(x^7)\right]^4\left[x\cos 2x\dfrac{d}{dx}(2x) + \sin 2x + 4\tan^3(x^7)\dfrac{d}{dx}\tan(x^7)\right]$

$= 5\left[x\sin 2x + \tan^4(x^7)\right]^4\left[2x\cos 2x + \sin 2x + 28x^6\tan^3(x^7)\sec^2(x^7)\right]$