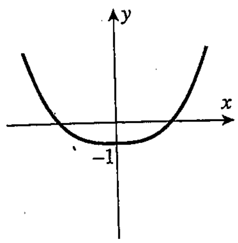
EXERCISE SET 3.2

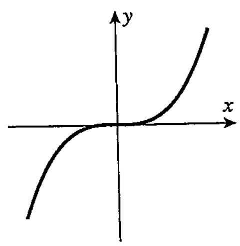
1. $f'(1) = 2,\ f'(3) = 0,\ f'(5) = -2,\ f'(6) = -1$

2. $f'(4) < f'(0) < f'(2) < 0 < f'(-3)$

3. **(b)** $m = f'(2) = 3$ **(c)** the same, $f'(2) = 3$

4. $m = \dfrac{2-(-1)}{1-(-1)} = \dfrac{3}{2}, y - 2 = m(x-1), y = \dfrac{3}{2}x + \dfrac{1}{2}$

5. 

6. 

7. $y - (-1) = 5(x-3),\ y = 5x - 16$

8. $y - 3 = -4(x+2),\ y = -4x - 5$

9. $f'(1) = \lim\limits_{h\to 0} \dfrac{f(1+h) - f(1)}{h} = \lim\limits_{h\to 0} \dfrac{2(1+h)^2 - 2\cdot 1^2}{h} = \lim\limits_{h\to 0} \dfrac{4h + 2h^2}{h} = 4;$
$y - 2 = 4(x-1), y = 4x - 2$

10. $f'(-1) = \lim\limits_{h\to 0} \dfrac{f((-1)+h) - f(-1)}{h} = \lim\limits_{h\to 0} \dfrac{1/(h-1)^2 - 1}{h} = \lim\limits_{h\to 0} \dfrac{1-(h-1)^2}{h(h-1)^2}$
$= \lim\limits_{h\to 0} \dfrac{2h - h^2}{h(h-1)^2} = \lim\limits_{h\to 0} \dfrac{2-h}{(h-1)^2} = 2;$
$y - 1 = 2(x+1), y = 2x + 3$

11. $f'(0) = \lim\limits_{h\to 0} \dfrac{f(0+h) - f(0)}{h} = \lim\limits_{h\to 0} \dfrac{h^3 - 0^3}{h} = \lim\limits_{h\to 0} h^2 = 0;$
$f'(0) = 0$ so $y - 0 = (0)(x-0),\ y = 0$

12. $f'(-1) = \lim\limits_{h\to 0} \dfrac{f(-1+h) - f(-1)}{h} = \lim\limits_{h\to 0} \dfrac{2(-1+h)^3 + 1 - (2(-1)^3 + 1)}{h} = \lim\limits_{h\to 0}(6 - 6h + 2h^2) = 6;$
$f(-1) = 2(-1)^3 + 1 = -1,\ f'(-1) = 6$ so $y + 1 = 6(x+1),\ y = 6x + 5$

13. $f'(8) = \lim\limits_{h\to 0} \dfrac{f(8+h) - f(8)}{h} = \lim\limits_{h\to 0} \dfrac{\sqrt{9+h} - 3}{h}$
$= \lim\limits_{h\to 0} \dfrac{\sqrt{9+h} - 3}{h} \dfrac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3} = \lim\limits_{h\to 0} \dfrac{h}{h(\sqrt{9+h} + 3)} = \dfrac{1}{6};$
$f(8) = \sqrt{8+1} = 3,\ f'(8) = \dfrac{1}{6}$ so $y - 3 = \dfrac{1}{6}(x-8),\ y = \dfrac{1}{6}x + \dfrac{5}{3}$

14. $$f'(4) = \lim_{h\to 0}\frac{f(4+h)-f(4)}{h} = \lim_{h\to 0}\frac{\sqrt{2(4+h)+1}-\sqrt{2\cdot 4+1}}{h}\frac{\sqrt{9+2h}+3}{\sqrt{9+2h}+3}$$
$$= \lim_{h\to 0}\frac{2h}{h(\sqrt{9+2h}+3)} = \lim_{h\to 0}\frac{2}{\sqrt{9+2h}+3} = \frac{1}{3}$$
$$f(4) = \sqrt{2(4)+1} = \sqrt{9} = 3, f'(4) = 1/3 \text{ so } y-3 = \frac{1}{3}(x-4), y = \frac{1}{3}x+\frac{5}{3}$$

15. $$f'(x) = \lim_{\Delta x\to 0}\frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x} = \lim_{\Delta x\to 0}\frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}$$
$$= \lim_{\Delta x\to 0}\frac{-\Delta x}{x\Delta x(x+\Delta x)} = \lim_{\Delta x\to 0}-\frac{1}{x(x+\Delta x)} = -\frac{1}{x^2}$$

16. $$f'(x) = \lim_{\Delta x\to 0}\frac{\frac{1}{(x+\Delta x)+1}-\frac{1}{x+1}}{\Delta x} = \lim_{\Delta x\to 0}\frac{\frac{(x+1)-(x+\Delta x+1)}{(x+1)(x+\Delta x+1)}}{\Delta x}$$
$$= \lim_{\Delta x\to 0}\frac{x+1-x-\Delta x-1}{\Delta x(x+1)(x+\Delta x+1)} = \lim_{\Delta x\to 0}\frac{-\Delta x}{\Delta x(x+1)(x+\Delta x+1)}$$
$$= \lim_{\Delta x\to 0}\frac{-1}{(x+1)(x+\Delta x+1)} = -\frac{1}{(x+1)^2}$$

17. $$f'(x) = \lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-(x+\Delta x)-(x^2-x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{2x\Delta x+\Delta x^2-\Delta x}{\Delta x}$$
$$= \lim_{\Delta x\to 0}(2x-1+\Delta x) = 2x-1$$

18. $$f'(x) = \lim_{\Delta x\to 0}\frac{(x+\Delta x)^4-x^4}{\Delta x} = \lim_{\Delta x\to 0}\frac{4x^3\Delta x+6x^2(\Delta x)^2+4x(\Delta x)^3+(\Delta x)^4}{\Delta x}$$
$$= \lim_{\Delta x\to 0}(4x^3+6x^2\Delta x+4x(\Delta x)^2+(\Delta x)^3) = 4x^3$$

19. $$f'(x) = \lim_{\Delta x\to 0}\frac{\frac{1}{\sqrt{x+\Delta x}}-\frac{1}{\sqrt{x}}}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sqrt{x}-\sqrt{x+\Delta x}}{\Delta x\sqrt{x}\sqrt{x+\Delta x}}$$
$$= \lim_{\Delta x\to 0}\frac{x-(x+\Delta x)}{\Delta x\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x}+\sqrt{x+\Delta x})} = \lim_{\Delta x\to 0}\frac{-1}{\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x}+\sqrt{x+\Delta x})} = -\frac{1}{2x^{3/2}}$$

20. $$f'(x) = \lim_{\Delta x\to 0}\frac{\frac{1}{\sqrt{x+\Delta x-1}}-\frac{1}{\sqrt{x-1}}}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sqrt{x-1}-\sqrt{x+\Delta x-1}}{\Delta x\sqrt{x-1}\sqrt{x+\Delta x-1}}\frac{\sqrt{x-1}+\sqrt{x+\Delta x-1}}{\sqrt{x-1}+\sqrt{x+\Delta x-1}}$$
$$= \lim_{\Delta x\to 0}\frac{-\Delta x}{\Delta x\sqrt{x-1}\sqrt{x+\Delta x-1}(\sqrt{x-1}+\sqrt{x+\Delta x-1})}$$
$$= \lim_{\Delta x\to 0}\frac{-1}{\sqrt{x-1}\sqrt{x+\Delta x-1}(\sqrt{x-1}+\sqrt{x+\Delta x-1})} = -\frac{1}{2(x-1)^{3/2}}$$