21. $f'(t) = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h} = \lim_{h\to 0} \frac{[4(t+h)^2+(t+h)]-[4t^2+t]}{h}$

$= \lim_{h\to 0} \frac{4t^2+8th+4h^2+t+h-4t^2-t}{h}$

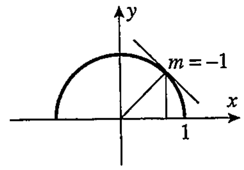
$= \lim_{h\to 0} \frac{8th+4h^2+h}{h} = \lim_{h\to 0}(8t+4h+1) = 8t+1$

22. $\frac{dV}{dr} = \lim_{h\to 0} \frac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3}{h} = \lim_{h\to 0} \frac{\frac{4}{3}\pi(r^3+3r^2h+3rh^2+h^3-r^3)}{h}$

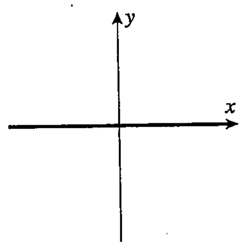
$= \lim_{h\to 0} \frac{4}{3}\pi(3r^2+3rh+h^2) = 4\pi r^2$

23. (a) D (b) F (c) B (d) C (e) A (f) E

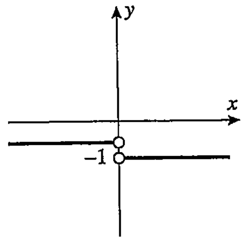
24. The point $(\sqrt{2}/2, \sqrt{2}/2)$ lies on the line $y = x$.



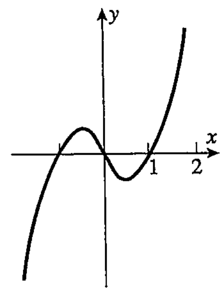
25. (a)



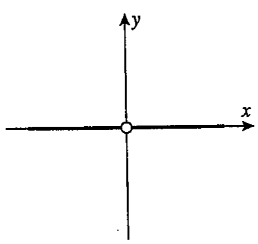
(b)



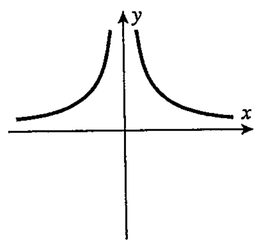
(c)



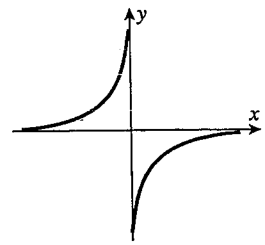
26. (a)



(b)



(c)



27. (a) $f(x) = \sqrt{x}$ and $a = 1$ (b) $f(x) = x^2$ and $a = 3$

28. (a) $f(x) = \cos x$ and $a = \pi$ (b) $f(x) = x^7$ and $a = 1$

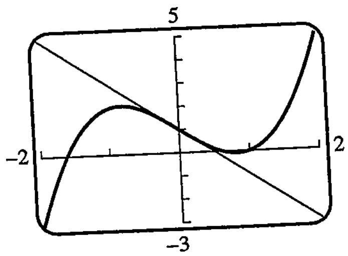
29. $\frac{dy}{dx} = \lim_{h\to 0} \frac{(1-(x+h)^2)-(1-x^2)}{h} = \lim_{h\to 0} \frac{-2xh-h^2}{h} = \lim_{h\to 0} -2(x+h) = -2x$,

and $\left.\frac{dy}{dx}\right|_{x=1} = -2$

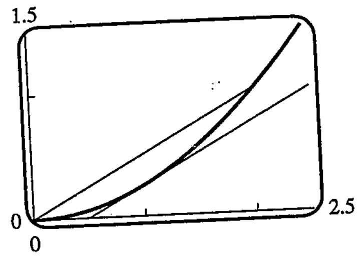
30. $\dfrac{dy}{dx} = \lim_{h\to 0} \dfrac{\dfrac{x+2+h}{x+h} - \dfrac{x+2}{x}}{h} = \lim_{h\to 0} \dfrac{x(x+2+h)-(x+2)(x+h)}{hx(x+h)} = \lim_{h\to 0} \dfrac{-2}{x(x+h)} = \dfrac{-2}{x^2}$,

and thus $\left.\dfrac{dy}{dx}\right|_{x=-2} = -\dfrac{1}{2}$

31. $y = -2x + 1$



32.



33. (b)

h	0.5	0.1	0.01	0.001	0.0001	0.00001
$(f(1+h)-f(1))/h$	1.6569	1.4355	1.3911	1.3868	1.3863	1.3863

34. (b)

h	0.5	0.1	0.01	0.001	0.0001	0.00001
$(f(1+h)-f(1))/h$	0.50489	0.67060	0.70356	0.70675	0.70707	0.70710

35. (a) dollars/ft

(b) As you go deeper the price per foot may increase dramatically, so $f'(x)$ is roughly the price per additional foot.

(c) If each additional foot costs extra money (this is to be expected) then $f'(x)$ remains positive.

(d) From the approximation $1000 = f'(300) \approx \dfrac{f(301)-f(300)}{301-300}$ we see that $f(301) \approx f(300) + 1000$, so the extra foot will cost around \$1000.

36. (a) gallons/dollar

(b) The increase in the amount of paint that would be sold for one extra dollar.

(c) It should be negative since an increase in the price of paint would decrease the amount of paint sold.

(d) From $-100 = f'(10) \approx \dfrac{f(11)-f(10)}{11-10}$ we see that $f(11) \approx f(10) - 100$, so an increase of one dollar would decrease the amount of paint sold by around 100 gallons.

37. (a) $F \approx 200$ lb, $dF/d\theta \approx 50$ lb/rad (b) $\mu = (dF/d\theta)/F \approx 50/200 = 0.25$

38. The derivative at time $t = 100$ of the velocity with respect to time is equal to the slope of the tangent line, which is approximately $m \approx \dfrac{10272-0}{120-40} = 128.4\ \text{ft/s}^2$. Thus the mass is approximately $M(100) \approx \dfrac{T}{dv/dt} = \dfrac{7680982}{128.4} \approx 59820.73\ \text{lb/ft/s}^2$.

39. (a) $T \approx 115°\text{F}$, $dT/dt \approx -3.35°\text{F/min}$

(b) $k = (dT/dt)/(T - T_0) \approx (-3.35)/(115-75) = -0.084$