EXERCISE SET 2.6

1. none

2. $x = \pi$

3. $n\pi, n = 0, \pm 1, \pm 2, \ldots$

4. $x = n\pi + \pi/2,\ n = 0, \pm 1, \pm 2, \ldots$

5. $x = n\pi,\ n = 0, \pm 1, \pm 2, \ldots$

6. none

7. $2n\pi + \pi/6, 2n\pi + 5\pi/6,\ n = 0, \pm 1, \pm 2, \ldots$

8. $x = n\pi + \pi/2,\ n = 0, \pm 1, \pm 2, \ldots$

9. $[-1, 1]$

10. $(-\infty, -1] \cup [1, \infty)$

11. $(0, 3) \cup (3, +\infty)$

12. $(-\infty, 0) \cup (0, +\infty)$, and if f is defined to be e at $x = 0$, then continuous for all x

13. $(-\infty, -1] \cup [1, \infty)$

14. $(-3, 0) \cup (0, \infty)$

15. **(a)** $\sin x, x^3 + 7x + 1$ **(b)** $|x|, \sin x$ **(c)** $x^3, \cos x, x + 1$
(d) $\sqrt{x}, 3 + x, \sin x, 2x$ **(e)** $\sin x, \sin x$ **(f)** $x^5 - 2x^3 + 1, \cos x$

16. **(a)** Use Theorem 2.5.6. **(b)** $g(x) = \cos x,\ g(x) = \dfrac{1}{x^2 + 1},\ g(x) = x^2 + 1$

17. $\cos\left(\lim_{x \to +\infty} \dfrac{1}{x}\right) = \cos 0 = 1$

18. $\sin\left(\lim_{x \to +\infty} \dfrac{\pi x}{2 - 3x}\right) = \sin\left(-\dfrac{\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$

19. $\sin^{-1}\left(\lim_{x \to +\infty} \dfrac{x}{1 - 2x}\right) = \sin^{-1}\left(-\dfrac{1}{2}\right) = -\dfrac{\pi}{6}$

20. $\ln\left(\lim_{x\to+\infty}\frac{x+1}{x}\right)=\ln(1)=0$

21. $3\lim_{\theta\to 0}\frac{\sin 3\theta}{3\theta}=3$

22. $\frac{1}{2}\lim_{h\to 0}\frac{\sin h}{h}=\frac{1}{2}$

23. $\left(\lim_{\theta\to 0^+}\frac{1}{\theta}\right)\lim_{\theta\to 0^+}\frac{\sin\theta}{\theta}=+\infty$

24. $\left(\lim_{\theta\to 0}\sin\theta\right)\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=0$

25. $\frac{\tan 7x}{\sin 3x}=\frac{7}{3\cos 7x}\frac{\sin 7x}{7x}\frac{3x}{\sin 3x}$ so $\lim_{x\to 0}\frac{\tan 7x}{\sin 3x}=\frac{7}{3(1)}(1)(1)=\frac{7}{3}$

26. $\frac{\sin 6x}{\sin 8x}=\frac{6}{8}\frac{\sin 6x}{6x}\frac{8x}{\sin 8x}$, so $\lim_{x\to 0}\frac{\sin 6x}{\sin 8x}=\frac{6}{8}=\frac{3}{4}$

27. $\frac{1}{5}\lim_{x\to 0^+}\sqrt{x}\lim_{x\to 0^+}\frac{\sin x}{x}=0$

28. $\frac{1}{3}\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2=\frac{1}{3}$

29. $\left(\lim_{x\to 0}x\right)\left(\lim_{x\to 0}\frac{\sin x^2}{x^2}\right)=0$

30. $\frac{\sin h}{1-\cos h}=\frac{\sin h}{1-\cos h}\frac{1+\cos h}{1+\cos h}=\frac{\sin h(1+\cos h)}{1-\cos^2 h}=\frac{1+\cos h}{\sin h}$; no limit

31. $\frac{t^2}{1-\cos^2 t}=\left(\frac{t}{\sin t}\right)^2$, so $\lim_{t\to 0}\frac{t^2}{1-\cos^2 t}=1$

32. $\cos(\frac{1}{2}\pi-x)=\sin(\frac{1}{2}\pi)\sin x=\sin x$, so $\lim_{x\to 0}\frac{x}{\cos\left(\frac{1}{2}\pi-x\right)}=1$

33. $\frac{\theta^2}{1-\cos\theta}\frac{1+\cos\theta}{1+\cos\theta}=\frac{\theta^2(1+\cos\theta)}{1-\cos^2\theta}=\left(\frac{\theta}{\sin\theta}\right)^2(1+\cos\theta)$ so $\lim_{\theta\to 0}\frac{\theta^2}{1-\cos\theta}=(1)^2 2=2$

34. $\frac{1-\cos 3h}{\cos^2 5h-1}\frac{1+\cos 3h}{1+\cos 3h}=\frac{\sin^2 3h}{-\sin^2 5h}\frac{1}{1+\cos 3h}$, so

$$\lim_{x\to 0}\frac{1-\cos 3h}{\cos^2 5h-1}=\lim_{x\to 0}\frac{\sin^2 3h}{-\sin^2 5h}\frac{1}{1+\cos 3h}=-\left(\frac{3}{5}\right)^2\frac{1}{2}=-\frac{9}{50}$$

35. $\lim_{x\to 0^+}\sin\left(\frac{1}{x}\right)=\lim_{t\to+\infty}\sin t$; limit does not exist

36. $\lim_{x\to 0}x-3\lim_{x\to 0}\frac{\sin x}{x}=-3$

37. $\frac{2-\cos 3x-\cos 4x}{x}=\frac{1-\cos 3x}{x}+\frac{1-\cos 4x}{x}$. Note that

$\frac{1-\cos 3x}{x}=\frac{1-\cos 3x}{x}\frac{1+\cos 3x}{1+\cos 3x}=\frac{\sin^2 3x}{x(1+\cos 3x)}=\frac{\sin 3x}{x}\frac{\sin 3x}{1+\cos 3x}$. Thus

$$\lim_{x\to 0}\frac{2-\cos 3x-\cos 4x}{x}=\lim_{x\to 0}\frac{\sin 3x}{x}\frac{\sin 3x}{1+\cos 3x}+\lim_{x\to 0}\frac{\sin 4x}{x}\frac{\sin 4x}{1+\cos 4x}=0+0=0$$

38. $\dfrac{\tan 3x^2+\sin^2 5x}{x^2}=\dfrac{3}{\cos 3x^2}\dfrac{\sin 3x^2}{3x^2}+5^2\dfrac{\sin^2 5x}{(5x)^2}$, so

$$\text{limit}=\lim_{x\to 0}\frac{3}{\cos 3x^2}\lim_{x\to 0}\frac{\sin 3x^2}{3x^2}+25\lim_{x\to 0}\left(\frac{\sin 5x}{5x}\right)^2=3+25=28$$

39. a/b

40. k^2s

41.

5.1	5.01	5.001	5.0001	5.00001	4.9	4.99	4.999	4.9999	4.99999
0.098845	0.099898	0.99990	0.099999	0.100000	0.10084	0.10010	0.10001	0.10000	0.10000

The limit is 0.1.

42.

2.1	2.01	2.001	2.0001	2.00001	1.9	1.99	1.999	1.9999	1.99999
0.484559	0.498720	0.499875	0.499987	0.499999	0.509409	0.501220	0.500125	0.500012	0.500001

The limit is 0.5.

43.

−1.9	−1.99	−1.999	−1.9999	−1.99999	−2.1	−2.01	−2.001	−2.0001	−2.00001
−0.898785	−0.989984	−0.999000	−0.999900	−0.999990	−1.097783	−1.009983	−1.001000	−1.000100	−1.000010

The limit is −1.

44.

−0.9	−0.99	−0.999	−0.9999	−0.99999	−1.1	−1.01	−1.001	−1.0001	−1.00001
0.405086	0.340050	0.334001	0.333400	0.333340	0.271536	0.326717	0.332667	0.333267	0.333327

The limit is 1/3.

45. Since $\lim_{x\to 0}\sin(1/x)$ does not exist, no conclusions can be drawn.

46. $k=f(0)=\lim_{x\to 0}\dfrac{\sin 3x}{x}=3\lim_{x\to 0}\dfrac{\sin 3x}{3x}=3$, so $k=3$

47. $\lim_{x\to 0^-}f(x)=k\lim_{x\to 0}\dfrac{\sin kx}{kx\cos kx}=k$, $\lim_{x\to 0^+}f(x)=2k^2$, so $k=2k^2$, $k=\dfrac{1}{2}$

48. No; $\sin x/|x|$ has unequal one-sided limits.

49. (a) $\lim_{t\to 0^+}\dfrac{\sin t}{t}=1$

(b) $\lim_{t\to 0^-}\dfrac{1-\cos t}{t}=0$ (Theorem 2.6.4)

(c) $\sin(\pi-t)=\sin t$, so $\lim_{x\to\pi}\dfrac{\pi-x}{\sin x}=\lim_{t\to 0}\dfrac{t}{\sin t}=1$

50. $\cos\left(\dfrac{\pi}{2}-t\right)=\sin t$, so $\lim_{x\to 2}\dfrac{\cos(\pi/x)}{x-2}=\lim_{t\to 0}\dfrac{(\pi-2t)\sin t}{4t}=\lim_{t\to 0}\dfrac{\pi-2t}{4}\lim_{t\to 0}\dfrac{\sin t}{t}=\dfrac{\pi}{4}$

51. $t=x-1$; $\sin(\pi x)=\sin(\pi t+\pi)=-\sin\pi t$; and $\lim_{x\to 1}\dfrac{\sin(\pi x)}{x-1}=-\lim_{t\to 0}\dfrac{\sin\pi t}{t}=-\pi$

52. $t=x-\pi/4$; $\tan x-1=\dfrac{2\sin t}{\cos t-\sin t}$; $\lim_{x\to\pi/4}\dfrac{\tan x-1}{x-\pi/4}=\lim_{t\to 0}\dfrac{2\sin t}{t(\cos t-\sin t)}=2$

53. $t=x-\pi/4$, $\dfrac{\cos x-\sin x}{x-\pi/4}=-\dfrac{\sqrt{2}\sin t}{t}$; $\lim_{x\to\pi/4}\dfrac{\cos x-\sin x}{x-\pi/4}=-\sqrt{2}\lim_{t\to 0}\dfrac{\sin t}{t}=-\sqrt{2}$